Using Mathematics to Design Costumes: Hat Making

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Abstract
Costuming and design for theatrical productions requires the creative use of mathematics and mathematical processes. To introduce students to the relationship between mathematics and art in costume design, mathematics educators collaborated with a costuming and design professor from the College Conservatory of Music to create this series of activities based on one expressive element – the hat. A central goal of the series of activities was to help students understand the role of mathematics in both the structural and aesthetic components of the design process. Mathematical Practices, Ratio and Proportion, and Geometry Common Core Standards are addressed.

When students ask “When are we ever going to use this?”, you will have an answer. Costuming and design! Drafting the patterns for costumes in play productions requires the creative use of mathematics, including spatial visualization, proportional reasoning, and measurement. Costume designers also need strong mathematical communication skills to convey the mathematical aspects of their creative process to their production team. This hat making activity is designed to introduce middle grades students to mathematical aspects of the costume design process so that they begin seeing mathematics as connected to future design careers.

Research on the connection between experiences in the arts and student achievement indicates that arts experiences for students contribute to: a) improved reading, language arts, and mathematics achievement; b) the development of thinking, social, and motivational skills; and c) improved attendance (Ruppert, 2005). For teachers, arts experiences promote innovative work, a positive professional culture, and effective instructional practice (Ruppert). In interviews, science educators claim that integrating science and the arts “lets students see how both of these have been and still are quests to examine and explain the world around us … Students see that curiosity, creativity, imagination and attention to detail are traits in common to artists/writers and scientists” (Liftig, 2010).

Designing and creating costumes for dramatic productions employs a variety of analytical skills for constructing meaning out of the playwrights’ texts. Before costume designers can draw renderings of costumes, they must first examine each character in a play and search out patterns and clues from the text that describe the character. Without specific detail in the prose, playwrights such as William Shakespeare gave costume designers lots of clues as to how characters, such as Puck in “A Midsummer Night’s Dream,” might look. The text of Shakespeare’s play makes it very clear that Puck is a creature that lives in a magical forest and serves the king of the fairies. However, specifies as to what type of warm weather garment a servant to a fairy king would wear are open to interpretation. It is at this precise moment in the creation of a costume’s design that logic and art must be combined. Costume designers must turn to their knowledge of world civilizations and fairy lore in search of elements that distinguish a servant from a king, while at the same time artistically suggesting that both the servant and the king are of a magical...
The color, size, texture, and movement of the entire costume from the hat to the shoes must all work together to express Puck’s social position and character in the mystical forest that Shakespeare envisioned. It is apparent that costume variations are potentially unlimited.

However, to narrow our learners’ (middle grade preservice teachers) focus, we created a lesson plan based on one very important and expressive element of costume design – the hat! The lesson invites learners to make connections between the mathematical and artistic elements of the design process. This lesson could be used for both preservice teachers and middle school students.

**Status, Style and Glamour of Hats**

The basic shape of the hat has not changed for hundreds of years. The first head covering was invented in order to protect people from the weather. This basic hat shape is sometimes called the crown (Figure 1). If wearers wanted to keep the sun or the rain out of their eyes, they added a brim. Historically, it did not take long for hats to evolve from purely functional forms to elaborate displays of fashion, power and wealth (Dial, 2002; Dreher, 1982; McDowell, 1992).

Royalty displayed their wealth and power through their extravagant use of precious metals and jewels in their crowns, while religious leaders often chose to display their high status by creating hats that rose towards the heavens (Figure 2B). For example, 15th Century Renaissance Ladies wore excessively tall hats called hennins, which were so difficult to balance they could not walk without the assistance of their ladies-in-waiting (Figure 2D). Later, during the 17th Century, Musketeers displayed their military pride and wealth by wearing hats with very large brims turned up and donned with tremendously expensive feathers (Figure 1A). Ironically, the large brims caused the hats to be blown off when they were riding their horses. Clearly, even for 18th Century Military Captains, practicality did not always win out over extravagant displays of military pride and power (Figure 1D).

Additionally, pride and the desire for wealth have not escaped the sports arena when it comes to the development of hats. The baseball cap (Figure 2) was no more than a simple functional hat used to keep the sun out of players’ eyes. Most historians attribute Babe Ruth with the beginning of America’s favorite pastime that coincided with the creation of the baseball cap as we know it today. Almost immediately, this simple functional hat rose to define the symbol of hope and American pride for working class people struggling to survive the great depression.

![Figure 1. Crown Hat Sketches (Truhart)](image)

**A central goal of this project is to help middle grade students understand the role of mathematics in both the structural and aesthetic components of the fashion design process by participating in a modified version of that process.**

![Figure 2. Baseball Cap Sketches (Truhart)](image)
As noted previously, costume designers use both analytical and artistic skills to define certain characters such as Puck, a Musketeer, or a famous baseball player by choosing specific details in the size, shape, color or decoration of a hat just like people have done to define themselves for centuries. What follows is our description of a design project focused on hats, hat making, and mathematics. A central goal of this project is to help middle grade students understand the role of mathematics in both the structural and aesthetic components of the fashion design process by participating in a modified version of that process. Below is an outline of the activities for the design project accompanied by a brief discussion of the strategies and thinking that preservice middle grades mathematics teachers used when we employed them in a methods course.

**Hat Making Project**

This design project is implemented in parts over the course of several days. Through these activities, we model key aspects of the design process - a cyclical process of initial planning, creation of a design, reflection, and revision. This design project addresses standards in both Arts and Mathematics education:

**National Standards for Arts Education** addressed:
- Visual Arts (Grades 5-8, Content Standard 6)
  Making connections between visual arts and other disciplines.
  Compare the characteristics of works in two or more art forms that share similar subject matter, historical periods, or cultural context.
  Describe ways in which the principles and subject matter of other disciplines taught in the school are interrelated with the visual arts. ([http://artsedge.kennedy-center.org/teach/standards/standards_58.cfm#04](http://artsedge.kennedy-center.org/teach/standards/standards_58.cfm#04))

**Common Core Standards for Mathematics (2010)** addressed:
- Mathematical Practice 4: Model with Mathematics–Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.
- Mathematical Practice 6: Attend to Precision–Mathematically proficient students try to communicate precisely to others.
- Grade 6 RP: Understand ratio concepts and use ratio reasoning to solve problems.
- Grade 7 RP: Analyze proportional relationships and use them to solve real-world and mathematical problems.
- Grade 7G: Draw, construct and describe geometrical figures and describe the relationships between them.
- Grade 8G: Understand congruence and similarity using physical models, transparencies, and geometry software. ([www.corestandards.org](http://www.corestandards.org))

**Part 1: Creating a Prototype**

Working in small groups, students choose one of the hat patterns (Figure 3: Basic Crown Hat or Figure 4: Baseball Cap) and use the pattern to create a hat that fits the head of one person in the group. The hat can be made using scrap paper, scissors, rulers, and tape. After the activity, pose the questions:

- What strategies and tools did you use to create your hat?
- Are the hats mathematically similar to the original pattern? Why or why not? How do you know?
- Are all of the hats created mathematically similar to each other? Why or why not?
- Did you make aesthetic adjustments to your hat? If so, how did you make these adjustments?
- What advice would you give to someone working to recreate a replica of your hat?
Preservice teachers in three groups who chose to make crown hats draped the paper around one preservice teacher’s head to make cylinders and then traced the circumferences of their cylinders to make the tops and the brims. They cut those hat parts out and taped them to the crown. Preservice teachers in two groups that chose to make baseball caps seemed to have more difficulty. One group cut eight triangular shapes that “looked as though they might fit” the circumference of a preservice teacher’s head but then had difficulty trying to decide how to put those triangular shapes together. At one point the shapes were arranged like this:

However, after much debate they rearranged the triangles like this:

With this arrangement they formed a cap and then struggled with how to make the hat bill. The first bill they made was too small. The second group used a “freehand” method to draw the triangular pieces so that they curved to fit the shape of one preservice teacher’s head and used “trial and error” (their words) to create the cap.

Once learners have been given the opportunity to create a hat of their own using a costume designer’s initial mock up, they begin to realize that the design process is not as simple as it might appear. In fact, we have used this activity with six classes of preservice teachers and one group of in-service teachers and, surprisingly, only two or three groups have ever taken a formalized approach using measurements, ratios, and proportions. Additionally, while a rough sketch of a crown hat may appear circular, the human head is not circular. Or, scaling a two-dimensional pattern for a three-dimensional model may result in unanticipated aesthetic changes that affect the desired look of the completed hat.

Part 2: Scaling Up a Pattern

Our next activity is designed to help learners connect proportional reasoning and measurement with the process of creating more precise mathematical instructions so that someone else—a theater company hat making team or a manufacturing company which will mass-produce the hat—can replicate “your” hat.

Using graph paper, learners work in pairs to scale up one piece of the baseball cap pattern by an assigned factor. Teachers may differentiate the project by giving different fractional values to different groups. Examples of student work samples are shown in Figure 5. The activity continues with a peer assessment in which learners are asked to evaluate the work product of another group (see Figure 6).
Our preservice teachers were generally quite accurate in scaling up the height and width of pattern pieces. However, they had more difficulty preserving the overall shape as they scaled up the curved parts of the pattern piece. In our whole class discussion, learners explained how they judged the quality of the scaled-up shape. Through both paper folding and mental visualization, learners noticed that portions of the original shapes had lines of reflectional symmetry that helped in determining if the shape of the piece was accurate. These lines are noted in blue in the Geometer’s Sketchpad drawing in Figure 7. Learners noted that if the blue lines are lines of symmetry for all or part of the shape in the original plan, then there should also be corresponding lines of symmetry in the scaled-up versions. Thus, we were able to discuss how one might use reflection to save time when checking if the shape of the enlarged pattern piece was similar to the shape of the original pattern piece. Discussion continued around the challenges of dilating shapes that are not simple polygons.

Conceptual understanding of proportion and similarity are mathematical goals of this activity. Within the broader project, this activity positions learners to view mathematical precision and communication as increasingly more important when you are creating directions for someone else (or a

You and your team are supervising a group of milliners in the costume department of a theatrical company. The milliners have scaled up pieces of a hat pattern that will be used to create hats for a musical. Fabric for the hats is expensive. Before production begins, you need to check the patterns for accuracy and provide feedback to the workers so that they can correct any problems with the patterns.

Check and comment on each of three areas:

1) Is the width of the pattern piece scaled up accurately? How can you tell?
2) Is the height of the pattern piece scaled up accurately? How can you tell?
3) Is the shape of the drawing preserved in the new drawing? What evidence supports your answer?
4) What suggestions for changes would you give?

Figure 6. Peer Assessment Prompt
Part 3: Analyzing the Structure of Different Hats

For the next part of this project, we address the need for learners to develop both the spatial visualization skills necessary for success in a variety of creative and artistic endeavors and to recognize those skills as mathematical. This activity is designed to help learners see the geometric structure in different hat designs and to provide an example of how that structure can be communicated pictorially to others.

Learners are given a handout (see Figure 8) and asked to: a) identify which hat diagram matches which hat, b) explain their reasoning in writing, and c) defend their choices in a whole class discussion.

Among our preservice teachers, there was initial disagreement in two areas. Discussion centered on whether Diagram A represented Picture 5 or Picture 3. There was additional discussion about Diagrams B and C and whether they corresponded to Pictures 1 or 2. Features of the diagrams that were highlighted in the discussions included the orientation, shape, and number of “petals.” Learners correctly noted that all hats in the sample were variations on the baseball cap pattern.

Given time constraints, we ended our project at this point. However, there are a number of ways teachers may extend this theme. For example, learners could be asked to create more precise mathematical patterns for constructing the hats they initially created in Part 1. These could be given to peers to recreate and the precision of the patterns evaluated. Alternatively, learners might be given time to design hat diagrams to match their own hat creations using traditional drawing tools or dynamic geometry software.

Conclusion

A central goal of this project is to help middle grade students understand the role of mathematics in both the structural and aesthetics components of the costuming and fashion design process by engaging
in a modified version of that process. By using connections to arts, a reluctant mathematics learner might be enticed to envision a career in which mathematics is a critical tool. When your students ask, “When are we going to use this?” after making hats, the answer will be more transparent. Costuming and design!!!

**References**


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