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AN APPLICATION OF THE BOX-COX TRANSFORMATION TO THE MONEY DEMAND IN COSTA RICA

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Abstract

This paper attempts to test different functional forms for the demand for money in Costa Rica, using the Box-Cox transformation. After an analysis of different statistics and economic theory criteria, the use of the log-linear specification is recommended, the closest theoretical functional form to the general transformation found. The introduction of some dynamics into the model is recommended, in order to correct for some autocorrelation problems.

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Norberto Zúñiga[1]

Introduction

The estimation of the demand for the money has been a central topic in Monetary Economics. Most of the existing theory has been based on the assumption that the demand for money is a stable function, which depends on a few explanatory variables, such as income and interest rates. The estimation of this function is important in developing countries, since most of their macroeconomic problems have been associated with the mismanagement of monetary policy. Furthermore, most of the stabilization programs supported by the International Monetary Fund have assumed the existence of a stable relationship of this monetary aggregate with its main determinants.

Although different theories explain the determination of the real demand for money, most of these approaches agree that the demand for money is basically influenced by a scale parameter, such as income, and by an opportunity cost variable, such as the interest rate. There is no theoretical support, however, for the choice of any particular functional form for this relationship. Most empirical studies have attempted to estimate three types of functional forms (linear, log-log, and log-linear), without providing any strong justification for their use.

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[1] The author is a doctoral student in Agricultural Economics at The Ohio State University. Comments from Claudio Gonzalez-Vega are gratefully acknowledged.

For these reasons, it would be interesting to use a flexible procedure that would make it possible to estimate a money demand without *a priori* assuming any functional form. The Box-Cox transformation can assist in this task of finding the functional form that better fits the data for the Costa Rican case.

I. Theoretical Background

Several approaches have emerged in the Economics literature that attempt to explain the reasons why people demand money. The basic question they try to answer is why people hold money while there are other kinds of assets that bring a tangible yield. The most relevant theories are the Quantitative, Keynesian, Neo-Keynesian theories, as well as the Reformulation of the Quantitative Theory. Despite some differences, most economists now agree that the demand for money can be seen as a demand for an asset that brings intangible yields and utility to its holders, due to its convenience and liquidity. Most economists also agree that the demand for money is determined by income and the opportunity cost of holding money. Accordingly, the demand for money is positively related to real income and negatively related to the nominal interest rate. In general,

$$m = f(y,r) + u \tag{1}$$

where: m is the real money demand, y is real income, r is the nominal interest rate, and u is a disturbance term.

Specific Functional Forms

Most empirical studies have attempted to estimate mainly three kinds of functional forms:

(*a*) Linear: $$m = a_1 + b_1 y + c_1 r + u \qquad (2)$$

(*b*) Log-log: $$\log m = a_2 + b_2 \log y + c_2 \log r + u \qquad (3)$$

(*c*) Log-linear: $$\log m = a_3 + b_3 \log y + c_3 r + u \qquad (4)$$

where all the variables are defined as above.

In the first functional form (linear), b_1 and c_1 are constant slope parameters corresponding to the income and interest rate variables. The elasticities associated with each variable are not fixed, however. They are given by $b_1(y/m)$ and $c_1(r/m)$ respectively.

In the second specification (log-log), the coefficients b_2 and c_2 provide a measure of the elasticities associated with the income and interest rate variables. These elasticities are assumed fixed through time.

The last model (log-linear) is a combination of the linear and the log-log specifications. Here, the elasticity associated with the income variable is assumed to be fixed (equal to b_3), while the interest rate elasticity is assumed to vary and is given by $c_3 r$. In this model, the parameter c_3 is known as the interest rate semi-elasticity of money demand, an important coefficient used extensively in macroeconomic models.

The last two specifications, and particularly the third one, have been strongly recommended in the Economics literature. There is, however, no theoretical justification for the use of any particular functional form. This is an empirical issue, that can be resolved through the use of flexible techniques that do not impose *a priori* any predetermined functional form and allow an analysis of the possibility of any specification being adequate. The Box-Cox transformation may be a useful devise to explore this issue.

Previous Empirical Results and Justification

The Box-Cox transformation technique may be useful in trying to estimate an appropriate functional form for demand for money in the Costa Rican case. A recent study by Soto and Cover (1988) attempted to estimate a money demand using a log-linear functional form. They found that the Chow test rejected the null hypothesis (at both 5 and 1 percent significance levels) that there is no structural change in the parameter estimates between 1964-1980 and 1980-1986.

Furthermore, Soto and Cover found that the estimation for both periods, using quarterly data, resulted in different income elasticities (0.43 for the first period and 0.52 for the second). This feature could not be captured by the annual model, since *a priori* it assumed a fixed income elasticity. This may be one of the reasons why their estimates are not totally free of autocorrelation problems.

Broader Perspective on the Demand for Money

In economies that are financially open, a measure of the yield of foreign assets, such as the foreign interest rate plus the expected rate of devaluation, may influence the domestic demand for money. As a country becomes more integrated to the international capital markets, new opportunities arise and one might expect a reallocation of the portfolios of assets of the domestic agents. In the Costa Rican case, however, this variable has generally neither been significant nor shown the correct sign (Soto and Cover, 1988). Hence, it will not be considered in the present estimations.

Since the demand for any good may be affected by population growth, this additional variable will be considered as a determinant of the demand for money. A positive relationship between the aggregate demand for money and the growth of population is expected.

Furthermore, in an attempt to relate monetary theory to consumer demand theory, where most conclusions are derived under the assumption of a "representative individual consumer," the regressions will be estimated in per capita terms. While avoiding some econometric problems, such as the presence of multicollinearity between income and population, this specification is consistent with the assumption that the demand for money can be derived from a maximization problem for individual economic agents (McCafferty, 1990:19-21).

In sum, three models will be estimated by using the Box-Cox transformation and the most popular functional forms:

$$(a) \qquad m* = a_1 + b_1 y* + c_1 r* + u \tag{5}$$

$$(b) \qquad m* = a_2 + b_2 y* + c_2 r* + d_2 n* + u \tag{6}$$

$$(c) \qquad (m/n)* = a_3 + b_3 (y/n)* + c_3 r* + u \tag{7}$$

where: m is the real money demand, measured as *M1* (currency plus checking deposits), deflated by the implicit deflator of GDP; y is the real Gross Domestic Product (GDP); r is the implicit deflator of GDP, used as a proxy for the interest rate variable[2]; n is population; and u is an error disturbance.

Box-Cox Transformation Technique

The Box-Cox transformation helps to find a more general functional form to represent the relationship between the dependent and explanatory variables. It is a useful technique for both discriminating among alternative functional forms and providing added flexibility in model specification. According to the Box-Cox transformation (Judge, 1988: 555-558), the variables with an asterisk are defined in general as:

$$x* = (X^1 - 1)/\lambda \qquad for\ \lambda \neq 0 \tag{8}$$

$$x* = \log x \qquad for\ \lambda = 0 \tag{9}$$

where λ represents a transformation parameter to be estimated. Thus, different values of λ lead to different functional specifications. This technique is very powerful because one may try the use of different combinations of the λ parameter. For example, one may allow λ to take different values just for the dependent variable; one may allow all dependent and explanatory variables to take the same λ; or one may allow λ to take different values for each variable. For instance, if $\lambda = 1$ or $\lambda = 0$ for all the variables, the specification is of the linear form or log-log function, respectively. If $\lambda_m = 0$, $\lambda_y = 0$, and $\lambda_p = 1$, one will have a log-linear functional form.

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[2] During most of the period analyzed, the interest rate was set by the monetary authority and not by the market. Instead of the interest rate, the implicit deflator of the GDP was used as a proxy for the opportunity cost of holding money stocks.

To estimate the parameters it is usually assumed that the error terms are independent, identically normally distributed with zero mean and constant variance σ^2. Maximum likelihood estimates can be found by:

(a) choosing a reasonable range of values for λ;

(b) using least squares to find the set of b and σ^2 for each value of λ; and

(c) choosing that set of estimates for which the concentrated likelihood function is a maximum.

According to Maddala (1989:178), this transformation has two main problems:

(a) the assumption that the error terms are independent, identically normally distributed with zero mean and constant variance does not seem to be reasonable, and

(b) the transformation of each variable itself imposes some constraints on the values that they can take, depending on the unknown λ. To deal with these problems and obtain more reliable estimates, one would test for heteroskedasticy, autocorrelation, and normality and attempt to correct for the problems observed.

Empirical Results

Using data obtained directly from the Costa Rican Central Bank for the 1957-1989 period and testing different specifications, the first model, in which real income and the implicit GDP deflator are the explanatory variables, was shown to be the most appropriate to describe the behavior of the real money demand in Costa Rica.[3] The findings discussed below are related to this model.

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[3] In general, in the second model, the population growth variable was not significant at any reasonable level, and in the third model the introduction of per capita variables reduced the significance of the opportunity cost variable.

By using the Shazam program, the Box-Cox transformation can be attempted under three different scenarios:

(a) applying the transformation to the dependent variable only;

(b) applying the transformation to all the variables and assuming the same λ for all of them; and

(c) applying the transformation to all the variables and assuming a different λ for each variable.

Since the regressions are not totally free of autocorrelation problems, the corresponding correction was attempted.

Table 1. Estimated Transformation, Autocorrelation Parameters and the Maximum Loglikelihood Function.

	Transformation Parameters			Autocorrelation Parameter	Value Likelihood Function
Model	λ_m	λ_y	λ_p	r	LL
Linear	1	1	1	0.27	-188.25
BCD	0.94	1	1	0.31	-188.21
BCDA	0.83	1	1	n.s.	-186.73
Log-log	0	0	0	0.35	-184.72
BCA	-0.19	-0.19	-0.19	0.34	-184.60
BCAA	-0.08	-0.08	-0.08	n.s.	-182.71
Log-linear	0	0	1	0.24	n.s.
BCF	-0.14	-0.12	1.71	0.15	-176.04

n.s.: not specified

Transforming the Dependent Variable Only

When only the dependent variable was allowed to assume any value for λ (BCD), the likelihood function was maximized when λ = 0.94, which is very close to unity. In fact, by applying the maximum likelihood ratio test one could not reject the null hypothesis that the specification of the money demand is of the linear form. When corrected for autocorrelation (BCDA), the new parameter estimate was $\lambda = 0.83$. Again, one could not reject the null hypothesis of a linear relationship.

Transforming All Variables: Assuming Equal λ

When λ was allowed to be the same for all the variables (BCA), the loglikelihood function was maximized when $\lambda = -0.19$, which is very close to zero. In fact, when the maximum likelihood ratio test was applied one could not reject the null hypothesis that the money demand is specified as a log-log function. When corrected for autocorrelation (BCAA), the value of $\lambda = -0.08$ was obtained, which is even closer to zero. Again, one could not reject the null hypothesis that the money demand can be specified as a log-log function.

Transforming All Variables: Assuming Different λ

A more interesting case is to allow λ to assume different values for each variable (BCF). In this case, $\lambda_m = -0.14$, $\lambda_y = -0.12$, and $\lambda_p = 1.71$. Here, I did not correct for autocorrelation, since this estimation was free of autocorrelation problems. This transformation showed the best performance in terms of all different statistics analyzed, along with the log-linear specification, as shown in Table 2. Actually, both look very similar in terms of performance.

In general all the estimates were highly significant and showed the expected sign (positive in the case of income, and negative in the case of the price index). All of the regressions associated with different specifications showed very high R-squared, as well. According to the Brusch-Pagan (B-P) test and the Jarque-Bera (J-B) test, the assumption of constant variance and normality in the error terms could not be rejected. However, according to the Durbin-Watson (D-W) statistic, just the BCF is totally free of auto-correlation problems, as stated above.

Table 2. Estimated Coefficients and some Statistics

	Coefficients			Statistics			
Model	Constant	y	p	R^2	D-W	B-P	J-B
Linear	-173.33	0.029	-12.867	0.979	1.468	5.262	0.452
	(40.50)	(0.01)	(2.06)				
BCD	-79.058	0.138	-8.705	0.979	1.436	5.073	0.505
	(26.70)	(0.01)	(1.36)				
Log-log	-4.383	1.307	-0.083	0.985	1.294	2.948	0.736
	(0.38)	(0.05)	(0.02)				
BCA	-4.053	1.862	-0.030	0.986	1.316	1.846	0.673
	(0.28)	(0.07)	(0.01)				
Log-linear	-3.810	1.238	-0.008	0.986	1.518	1.391	0.671
	(0.28)	(0.03)	(0.00)				
BCF	-35.738	8.833	-0.002	0.987	1.663*	0.185	0.658
	(1.13)	(0.21)	(0.00)				

Notes: Figures in parenthesis are standard errors.
All coefficient estimates are significant at the 5 percent level.
B-P: Brusch-Pagan test of heteroskedasticity was significant at the 5 percent level for all functional forms.
J-B: Jera-Bera test of normality was significant at the 5 percent level for all functional forms.
R^2: Adjusted R-squared.
* means significant at 5 percent level.

One has to decide which of the three results obtained suggests a more appropriate specification, since depending of the criteria used one can arrive at different conclusions. For instance, according to the first and second procedures, the linear and the log-log are the most convenient functional forms, respectively. According to the third criterion, however, the log-linear specification appears to be more attractive.

By analyzing some important statistics, one may think that the third procedure provides the most appropriate specification, since it gives the larger value of the loglikelihood function, the smallest autocorrelation coefficient,[4] the lowest degree of collinearity between the explanatory variables, the highest R-squared, and the smallest standard deviation in the adjusted regression. It also offers more flexibility than the other two in the transformation of the variables, since the value of λ is allowed to change for each variable.

Elasticity Analysis

A comparison of the coefficients from each functional form does not make much sense, since they are affected by the units in which the variables are measured. For this reason, the elasticities, computed at the means and for some years, from the linear, log-log, and log-linear specifications, provide a useful comparison.

In general, the income elasticity is very similar under each specification, both at the means and for some specific years. This coefficient is greater than unity for all of the three different functional forms, which means that as income increases, the real money demand increases more than proportionally. This result is consistent with those found for some developing countries (Soto and Cover, 1988; p. 25) and with the monetization process that Costa Rica has experienced. It is not in agreement, however, with the strong belief that there are economies of scale in money holdings, and from the influence of innovations and improvements in the computer and telecommunications area. As McCullum states, "if such considerations are important, as they almost certainly are, (the money demand) would not remain the same as time passes." To analyze this possibility in greater depth, a time

[4] This is the only specification that does not show any autocorrelation problems.

variable was included as an explanatory variable, following McCallum's recommendation. The coefficients obtained were not significant, although in most of the cases they had the expected sign (negative). This may reflect, however, the effects of increased financial repression since the late 1970s.

Table 3. Income and Price Elasticities of Money Demand in Costa Rica, Selected Years.

	Income Elasticities			Price Elasticities		
Year	Linear	Log-log	Log-linear	Linear	Log-log	Log-linear
1960	1.406	1.301	1.238	-0.026	-0.083	-0.008
1965	1.409	1.301	1.238	-0.021	-0.083	-0.008
1970	1.296	1.301	1.238	-0.017	-0.083	-0.010
1975	1.211	1.301	1.238	-0.022	-0.083	-0.019
1980	1.234	1.301	1.238	-0.034	-0.083	-0.041
1985	1.282	1.301	1.238	-0.163	-0.083	-0.171
1989	1.363	1.301	1.238	-0.258	-0.083	-0.306
At the means	1.277	1.301	1.238	-0.076	-0.083	-0.058

Regarding the interest rate elasticity, the results differed somewhat, depending on the specification chosen, but in all of them the coefficient was significant and had the expected sign (negative). While in the log-log specification one obtains a time-fixed coefficient (-0.83), in the linear and log-linear specifications this parameter has been increasing in absolute terms, although at the means the differences are smaller. This tendency may reflect the fact that the money demand is more volatile to changes in the opportunity cost variable and hence it is more sensitive to changes in the short run. Thus, as the inflation rate has been increasing, the demand for money has become more elastic, reflecting the absence of money illusion and the existence of profitable money substitutes. This fact may be better captured in a money demand specified as the BCF or as its closest formal specification, the log-linear form.

Conclusions and Recommendations

The careful analysis of statistical criteria along with theoretical considerations make it possible to recommend the estimation of the real money demand in Costa Rica according to the specification given by the log-linear functional form, which is the closest to the BCF.

From the statistical point of view, the application of the Box-Cox transformation under different procedures, along with the analysis of different kinds of statistics, such as the value of the likelihood function, the presence of autocorrelation, multicollinearity, variance of the estimated regression, and R-squared suggests the choice of the log-linear form as the most appropriate. From the economic theory point of view, this function also makes it possible to better capture the stable relationship between the demand for money and real income and the greater volatility of the relationship between this monetary aggregate and the opportunity cost variable.

Almost all of the different functional forms estimated show some autocorrelation problems, which in turn may suggest the possibility of improvement in the specification of the model. One possibility suggested by McCallum (1989:46) is to work with first-differenced data, since in this way the disturbance term is the difference of e_t rather than e_t itself, which in empirical work may imply a much smaller degree of serial correlation.

In addition to that, another approach that may be attempted is to incorporate some dynamics into the model, in the form of flow or stock adjustments. Under the first approach, the easiest way is to introduce as an additional explanatory variable the dependent variable lagged one period (m_{t-1}); under the second approach, one may introduce current independent variables lagged one period (i.e., y_{t-1} and p_{t-1} or r_{t-1}).

It may be better to run the model using quarterly data rather than annual observations. Besides offering more degrees of freedom and reducing some multicollinearity problems, this makes it possible to know how the money demand behaves in the short-run, in comparison with the long-run, and to capture in a more precise way the possibility of shifts in this relationship. Furthermore, as a measure of the opportunity cost, the use of the interest rate instead of the price index is recommended, particularly for the 1980s, when interest rates have been more freely set. In addition, broader measures of the definition of money demand, such as M_2 and M_3 might be considered. Actually, most of these recommendations have been undertaken by Soto and Cover (1988).

Finally, from the monetary policy point of view, the accurate estimation of the demand for money is relevant, since increases in the money supply will depend on the influence of the explanatory variables. For instance, if it is expected that the real GDP grows around 4 percent and prices increase around 15 percent[5], the monetary authority should increase the real money supply by around 4 or 0 percent, depending on whether one considers the values of the log-linear function at the means or for the last year, respectively. Even though this is a very risky example, it gives some insights about the careful management of monetary policy, to provide the necessary liquidity to economic agents in an environment of economic stability.

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[5] These figures are according to the behavior of these variables over the past few years.

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