

# High Accuracy Fragment Methodologies

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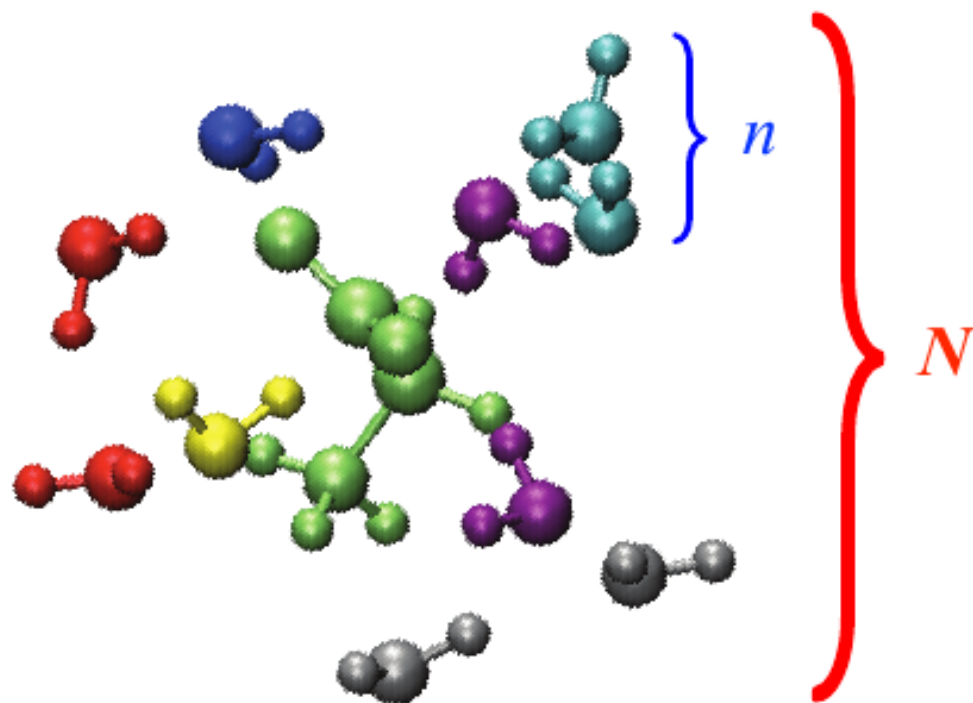
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# Fragment Methodologies

- ***Ab initio* methods scale non-linearly with system size**
- **Electronic matter is “near-sighted”**
  - **W. Kohn Phys. Rev. Lett. 1996**
- **Exploit this to reduce  $N$  to  $n$**



$$\mathcal{O}(N^p) \rightarrow N_{\text{frag}} \times \mathcal{O}(n^p)$$

# History

- **Reviews and classifications:**

- Gordon et al., Chem. Rev. 112, 632 (2011)
- Richard & Herbert, J. Chem. Phys. 137, 064113 (2012)

- **There are a lot of fragment-based methods in the literature. An incomplete list:**

- Many-body expansion of the correlation energy (Stoll & Preuß, 1977)
- Fragment Molecular Orbital (FMO) method (Kitaura, Fedorov et al., 1999)
- Molecular Fractionation with Conjugated Caps (MFCC) (Zhang et al., 2003)
- Multicentered QM:QM Method (Tschumper et al., 2003)
- Kernel Energy Method (KEM) (Huang, Massa et al., 2004)
- Systematic Molecular Fragmentation (SMF) method (Collins et al., 2005)
- Binary and Ternary Interaction Method (Hirata et al., 2005)
- Cardinality-Guided Molecular Tailoring Approach (CG-MTA) (Gadre et al., 2006)
- SCF for Molecular Interactions (SCF-MI) (Head-Gordon et al., 2006)
- Electrostatically-Embedded Many-Body (EE-MB) expansion (Truhlar et al., 2007)
- Generalized Energy-Based Fragmentation (GEBF) method (Li et al., 2007)
- Hybrid Many-Body Interaction (HMBI) method (Beran et al., 2009)
- Molecules-in-Molecules (MIM) method (Raghavachari et al., 2011)
- Variational Many-Body Expansion (Gao et al., 2012)

# Many-Body Expansion (MBE)

- **Possible to exactly expand the supersystem energy in terms of:**
  - **Fragment energies**
  - **Fragment interactions**

$$E_0 = \sum_{I=1}^N E_I + \sum_{I<J} {}^N C_2 \Delta E_{IJ} + \sum_{I<J<K} {}^N C_3 \Delta E_{IJK} + \cdots + \Delta E_{IJK\dots N}$$

$$\Delta E_{IJ} \equiv E_{IJ} - E_I - E_J$$

$$\Delta E_{IJK} \equiv E_{IJK} - \Delta E_{IJ} - \Delta E_{IK} - \Delta E_{JK} - E_I - E_J - E_K$$

# MBE Cont.

- **The MBE is:**
  - **Capable of chemical accuracy**
  - **Systematically improvable**
  - **“Embarrassingly parallel”**
- **Faster convergence if we include intersections**
  - **Adds (n+1)-body terms at n-body level**
    - Richard & Herbert, JCP 137, 064113 (2012)
    - Richard & Herbert, JCTC 9, 1408 (2013)

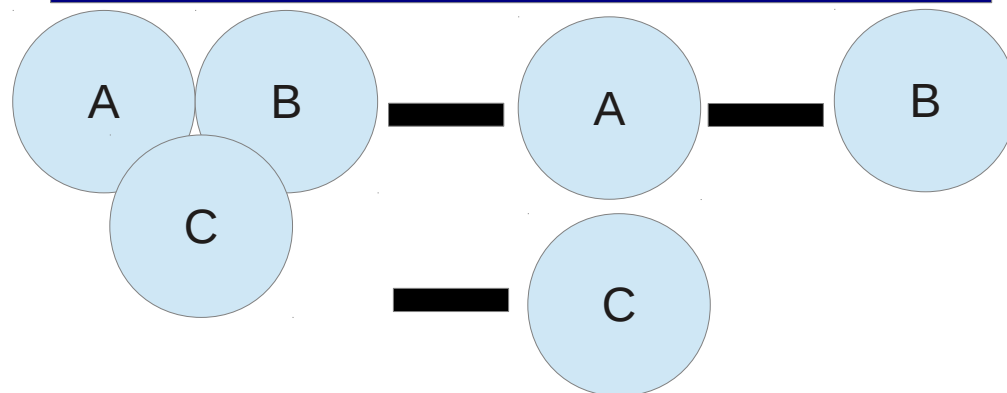
# MBE cont.

- **Currently limit ourselves to MBE**
- **Past MBE studies:**
  - **Replicate supersystem properties**
  - **Double  $\zeta$  basis sets**
- **Current Goal:**
  - **Experimental and/or high accuracy benchmarks**
    - **Approximate CCSD(T)/CBS**
      - **Basis-set superposition error (BSSE) is a problem**
      - **Slow convergence to CBS limit**

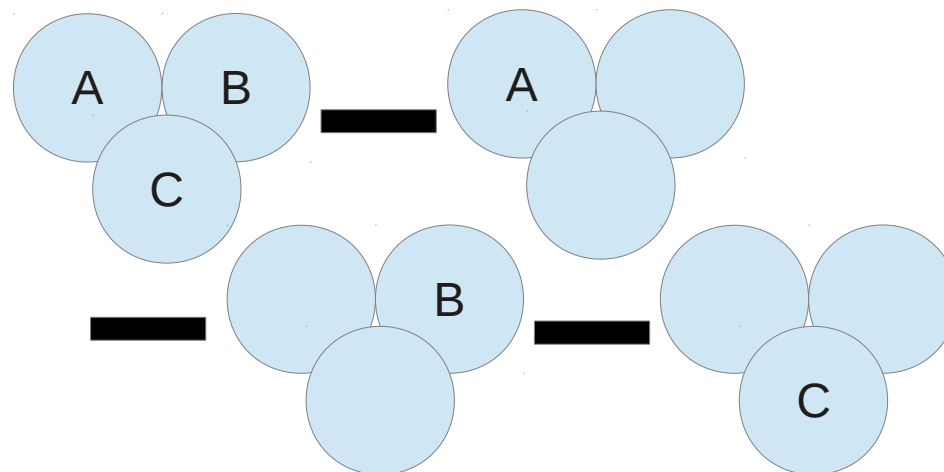
# Counterpoise (CP) Correction

- **BSSE is corrected by the CP correction**
  - Boys & Bernardi J. Mol. Phys. 1970
  - Corrections use the full basis-set
    - $N$  monomers need  $N+1$  full basis calculations

## Uncorrected Binding Energy



## BSSE Free Binding Energy



# Many-Body CP method [MBCPn]

- **Motivation: decrease time of a CP correction:**
  - **Fragment the CP corrections**
    - **Fragment the ghost atom + real system**
  - **Many MBE terms are 0**
    - **Consist of only ghost atoms**
  - **Embarrassingly parallel**



# MBCP(n)

$$\Delta E_I^{(2)} = -(N-1)E_I^I + \sum_{J \neq I}^N E_I^{IJ}$$

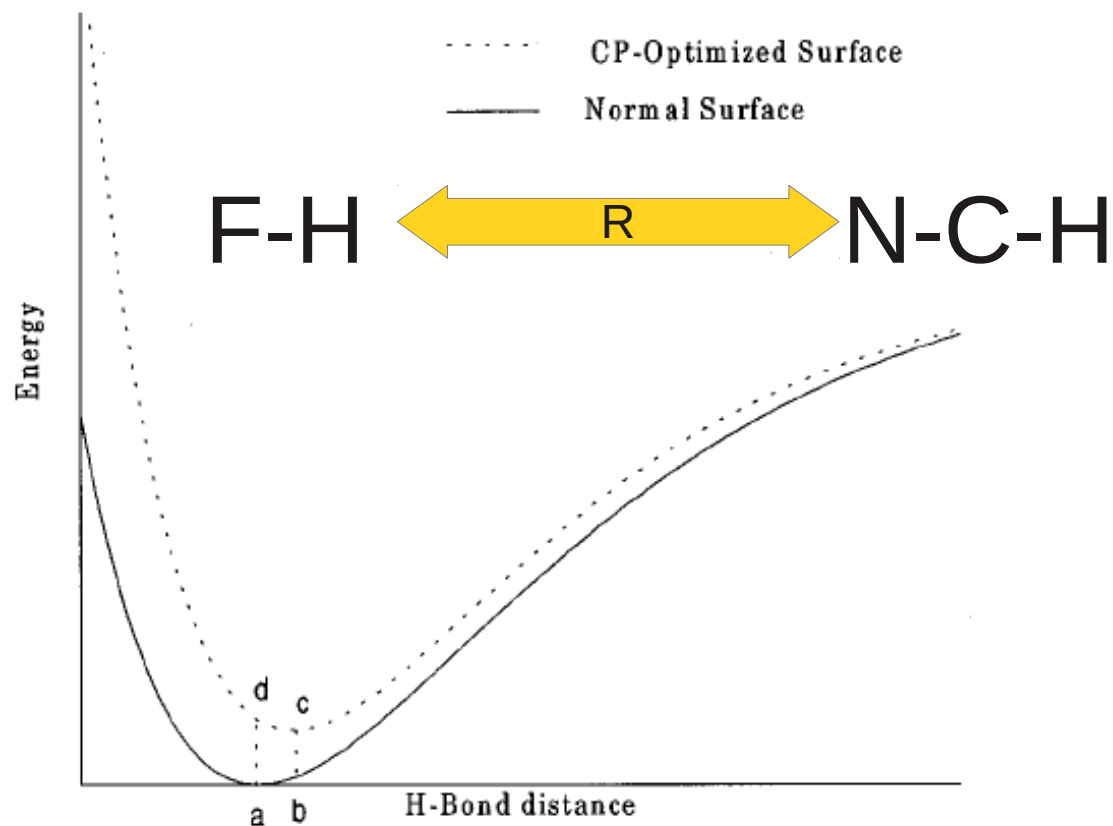
$$\Delta E_I^{(3)} = \frac{1}{2}N(N-1)E_I^I - (N-2) \sum_{J \neq I}^N E_I^{IJ} + \sum_{J \neq I}^N \sum_{\substack{K < J \\ K \neq I}}^N E_I^{IJK} .$$

- **Advantages:**

- **No full system basis-sets**
- **Easily extends to higher orders**

# Relevance to Spectroscopy

- Accurate spectra require accurate potential energy surfaces
  - Simon et al. JCP 105(1996) 11024

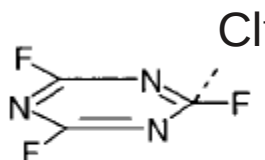


# Relevance Cont.

- Zero Point Energies

- Maçcal et al. JACS 124 (2002) 6274

species	$E_{(\text{MP2+ZPE})}$	$E_{(\text{MP2+ZPE-BSSE})}$
triazazine + chloride $\pi$ ( <b>3a</b> )	-8.3	-4.8
triazazine + fluoride $\pi$ ( <b>3b</b> )	-12.1 <sup>b</sup>	-9.2
triazazine + azide $\pi$ ( <b>3c</b> )	-7.6	-4.5
triazazine + fluoride "attack" ( <b>5b</b> )	-25.0	-18.5
triazazine + azide "attack" ( <b>5c</b> )	-9.0	-5.9
triazazine + chloride H-bond ( <b>7a</b> )	-9.6	-7.4
triazazine + fluoride H-bond ( <b>7b</b> )	-18.4	-16.5
triazazine + azide H-bond ( <b>7c</b> )	-8.0	-6.5
triazazine + azide "stack" ( <b>8</b> )	-10.6	-6.4
F <sub>3</sub> triazazine + chloride $\pi$ ( <b>4a</b> )	-19.7	-14.8
F <sub>3</sub> triazazine + fluoride $\pi$ ( <b>4b</b> )	-27.7 <sup>b</sup>	-24.0
F <sub>3</sub> triazazine + azide $\pi$ ( <b>4c</b> )	-17.9	-13.9
F <sub>3</sub> triazazine + chloride "attack" ( <b>6a</b> )	-22.1	-11.1
F <sub>3</sub> triazazine + fluoride "attack" ( <b>6b</b> )	-56.1	-48.8
F <sub>3</sub> triazazine + azide "attack" ( <b>6c</b> )	-27.0	-17.5
F <sub>3</sub> triazazine + azide "stack" ( <b>9</b> )	-20.0	-14.3

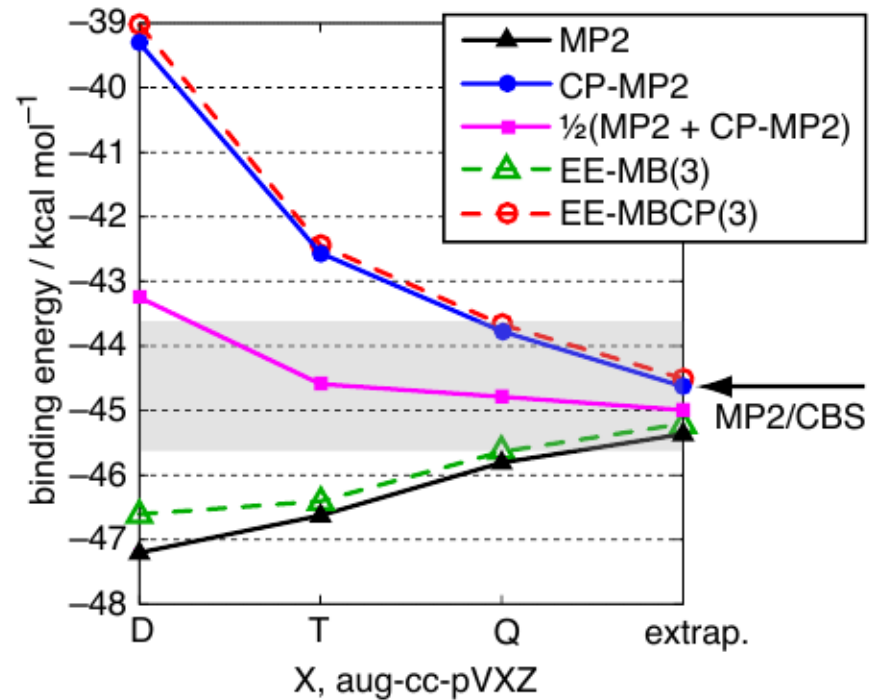


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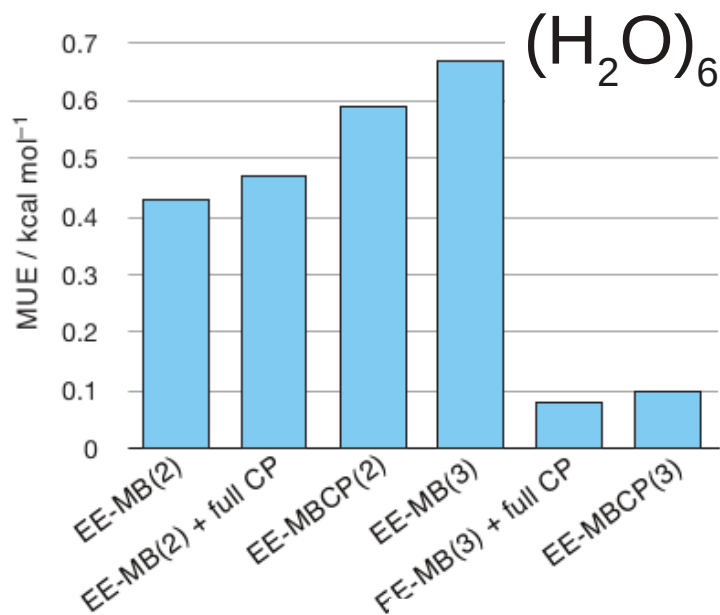
11  
kcal/mol

# Results

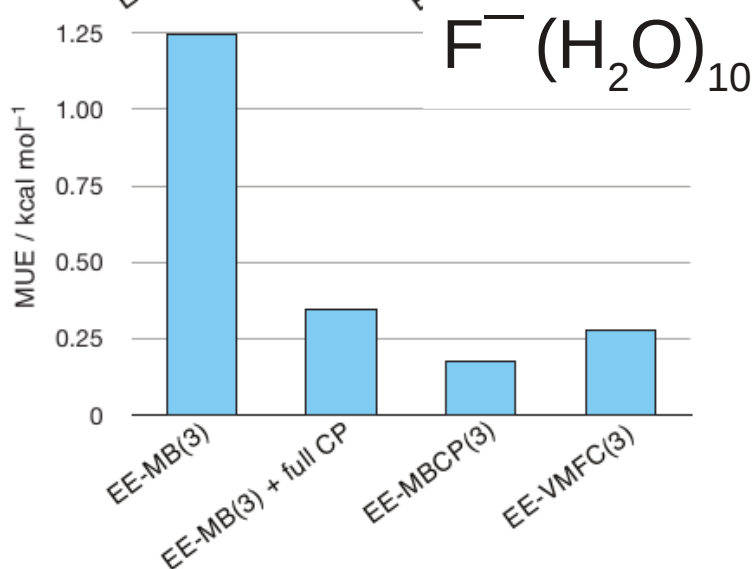
- $(\text{H}_2\text{O})_6$  is a well studied system
  - 8 structures
  - MP2-R12/CBS & CCSD(T)/CBS
    - Bates & Tschumper J Phys. Chem. A (2009)



# Results Cont.



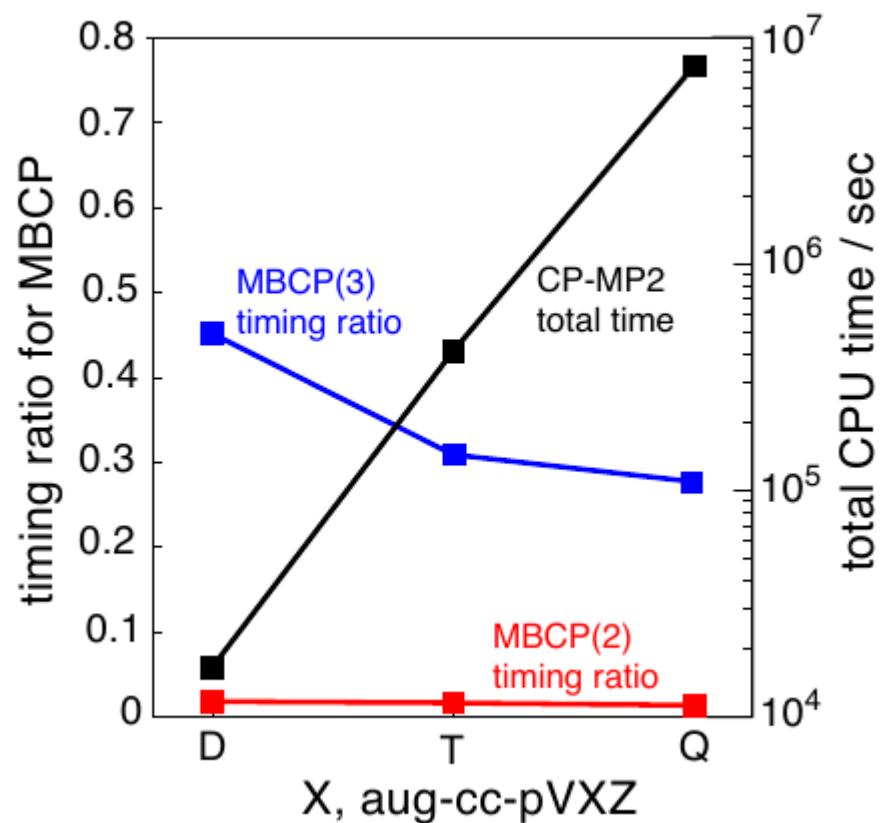
- **CP & MBCP(2) differ by ~0.1 kcal/mol**
- **CP & MBCP(3) negligibly different**



- **CP & MBCP(2) differ by ~0.6 kcal/mol**
- **CP & MBCP(3) differ by ~0.2 kcal/mol**

# Results Cont.

- **Time for  $F^-(H_2O)_{10}$**
- **Full CP ~100 days**
- **MBCP(3) ~1/3 time for quad  $\zeta$** 
  - **0.2 kcal/mol error**
- **MBCP(2) <10%**
  - **0.6 kcal/mol error**



# Triples Corrections

- **Triples correction using haTZ**
  - **aug-cc-pVTZ on O**
  - **cc-pVTZ on H**
- **2B: <0.1 kcal/mol**
- **3B: nearly exact**

Isomer	$\delta_{\text{CCSD(T)}}^b$	error <sup>c</sup>	
		EE-MB(2)	EE-MB(3)
Bag	0.33	-0.08	0.00
Boat1	0.53	0.04	0.01
Boat2	0.51	0.04	0.01
Book1	0.33	-0.04	0.00
Book2	0.33	-0.04	0.01
Cage	0.13	-0.20	0.00
Cyclic	0.53	0.05	0.01
Prism	-0.06	-0.20	0.00
MUE <sup>d</sup>	—	0.09	0.00

# Conclusions

- **Introduced an approximate CP correction**
  - **MBCP( $n$ )**
  - **No full basis calculations required**
  - **Accuracy can be increase by increasing  $n$**
  - **$n=3$  truncation accuracy:**
    - **Water hexamer: almost same as full CP**
    - **Fluoride cluster:  $< 0.2$  kcal/mol error**
- **Fragment methods are capable of chemical accuracy!!!**



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