The Limits of Multiplexing

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What are the limits to multiplexing?

- Physical limits to frequency multiplexing – Dynamic Range
  - Digitizer Effective Number of Bits (ENOB)
  - Signal-to-Noise Ratio (SNR)
  - MPDV Dynamic Range
  - Sources of noise due to optical amplification

- Physical limits to time multiplexing – Coherency
  - Degradation of beat signal visibility due to laser coherence and fiber-optic affects
  - Some preliminary laboratory measurements

- Practical constraints to large channel count experiments
  - The ‘little things’: fibers, connectors, polishing, cleaning
  - Data Assurance Methods and Tools: Transmissions & Optical Back Reflection (OBR) measurements
  - Cross-talk in many point experiments (discussion)
Frequency Multiplexing – How Deep can we go?

Sources that limit frequency Multiplexing

- Recording. Digitizer Effective Number of Bits limitation.
- Detection. Photo-diode noise floor limits ... SNR (see Rutkowski Report, will not discuss today).
- Optical. Optical amplifier noise can limit SNR
Frequency Multiplexing – Digitizer Effective Bits & SNR

Oscillator (sine wave) Electrical or optically generated

Digitizer

FFT

SNR

FFT of 1 GHz Oscillator

Noise Floor is a function of FFT window length = N

SNR = 6.02E + 1.76 + 20 log (2A/V) dB

E = effective bits for digitizer, V = full scale range, A = RMS amplitude of applied signal

See Wiley Encyclopedia of Electrical and Electronics Engineering, Vol. 18, J. Blair
Limits of Frequency Multiplexing – Digitizer SNR

- SNR: Frequency Domain
- SNR: Time Domain, Calculated

Approx Visual Limit of Spectra

Tek DPO 72004: Sine Wave Testing Electrical Signal (ENOB = 5.5)

27 dB SNR Increase from FFT N=1024
(\(\Delta \text{SNR} = 10 \log (N/2)\))

\[
\text{SNR}(t) = 6.02 \times 5.5 + 1.76 + 20 \log(2A/V)
\]

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Limits of Frequency Multiplexing – SNR for Electrically vs. Optically Generated Signals

Optically generated Beat Signal
\[ P_1 = P_2 = -10 \text{ dBm} \]
No EDFA pre-amp

\[ \text{SNR} = 6.02 \times 5.7 + 1.76 + 20 \log(2A/V_{FS}) \]
- Elec. SNR (avg noise floor, \(N=1024\))
- Elec. SNR (above noise floor, \(N=1024\))
- Optical SNR (above noise floor, \(N=1024\))
Limits of Frequency Multiplexing – MPDV Dynamic Range

Signal Optical Power, No EDFA (dBm)

-60 -50 -40 -30 -20 -10

MPDV Dynamic Range
Electrical & Optical Signals
CW (steady state), Power\text{\_local\_osc} = -10\,\text{dBm}

2X Mux
4X Mux

Visual Limit of Spectra

~ 40 dB Dynamic Range

\[ \text{SNR(t)} = 6.02 \times 5.7 + 1.76 + 20\log(2A/V_{FS}) \]

\( \times \) Optical Signal, No EDFA
(above noise floor, \( N=1024 \))

Fraction Full Scale Range of Scope \( \left( V_{pe}/V_{FS} \right) \)
Limits of Frequency Multiplexing – Noise from Optical Amplification

1. Use of Erbium Doped Fiber Pre-Amp generates amplified spontaneous emission (ASE) which beats with signal and local oscillator. G=Gain

2. ASE affects on SNR:
   - Local Oscillator (LO) – ASE
   - Signal - ASE
   - ASE – ASE (we usually neglect)

\[ I_{\text{sig}}^2 = 2\eta^2 G I_{s} I_{LO} \text{, where } I_{\text{sig}} = \frac{eP_s}{h\nu_s} \]

Noise Power Spectral Density

\[ \sigma^2_{LO-ASE} = 4\eta^2 I_{LO} I_{ASE} \frac{B_e}{B_o} \]

\[ \sigma^2_{s-ASE} = 4\eta^2 G I_{s} I_{ASE} \frac{B_e}{B_o} \]

LO – ASE noise usually dominates for typical conditions

Example: LO-ASE Noise \( \approx \) Miteq Noise Floor when
- Pre-amp, Gain \( \sim \) 23 dB
- ASE filtered, 200 GHz bandpass
- Power (local osc) \( \sim \) 100 microwatts
Limits of Frequency Multiplexing – EDFA Affects on SNR

Configuration: EDFA Pre-Amplification of optical signal returned from target surface

SNR (FFT, N=1024) vs Optical Power
1 GHz Heterodyne, CW Signal
EDFA Gain = 20 dB, P(Osc) = -10 dBm

Signal Optical Power, EDFA Input (dBm)

Signal Optical Power, No EDFA (dBm)
Time Multiplexing: How Deep can we go?

\[ I(\tau) = I_1 + I_2 + 2\sqrt{I_1 I_2} \ V(\tau) \ \cos \phi \]

\[ V(\tau) = \text{Fringe Visibility} \]
\[ 0 < V(\tau) < 1 \]

- Visibility is a function of laser linewidth $\Delta \nu$ (or equivalently temporal coherence).
- Temporal coherence characterized by coherence time $\tau_c \sim \alpha*(1/\Delta \nu)$ or equivalently coherence length $\ell_c = c \ \tau_c / n$
  
  $\alpha$ is multiplicative constant dependent on spectral line-shape (e.g. Gaussian, Lorentzian etc.)
- Example: $\Delta \nu = 15$ kHz linewidth, $n=1.47$ and $\alpha = 1$, then $\ell_c = 13.6$ km
- Linewidth is also degraded in fiber due to polarization mode dispersion (PMD)
**Time Multiplexing: Laboratory Measurements**

- **Laser 1**
  - Maintain constant power into Miteq photo-receiver.
  - Measure beat amplitude and spectrogram
  - Vary amount of fiber-optic time delay $\tau$.

- **Laser 1'**
  - 50:50 variable optical attenuator
  - 50:50 Power monitor

- **Fiber-optic delay**
  - $\tau = \text{round-trip total delay}$

- **Gold fiber-optic retro-reflector**

- **Photo-diode & Scope or Spectrum Analyzer**
- **Polarimeter**

**Work in Progress**

- No measureable effect ($V_{rms}$) for $\tau < 350 \mu s$ (70 km). Visibility and DOP $\approx 1$, but amplitude modulations consistent with polarization fluctuations evident for delays $\tau > \tau_c$. 

**Note:**
- Time Multiplexing: PDV Workshop, SNL, October 22-23
Limits of Multiplexing – Practical Consideration

Eight is the new one ... statistics will catch up with you. Many more fibers and connectors.

- QA is necessary, automation is desirable
  - fibers, connectors, polishing, cleaning.

- MPDV Data Assurance: Methods and Tools.
  - Fiber Transmissions measurements
  - Optical Back Reflection (OBR) measurements
  - LUNA measurements

- Cost and Risk
  - Data risk ... how many eggs in the basket?

Special Case for Off-Line Discussions:
Many channels ( > 100) in geometries conducive to cross-talk ... what to do?