# Wavelet analysis of heterodyne velocimetry (pdv) signals

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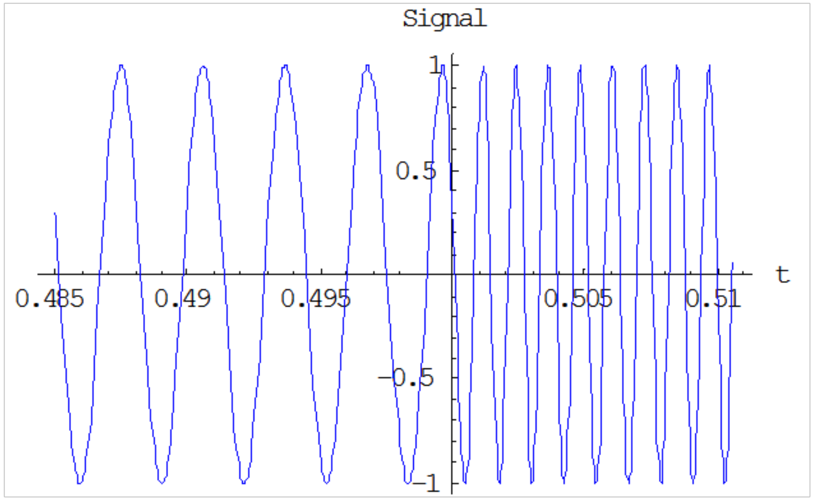
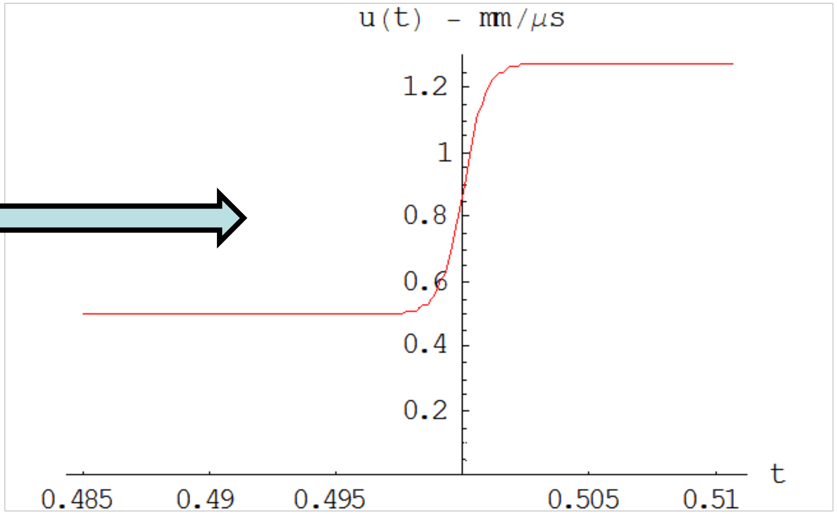
Thanks to

Dave Holtkamp, Brian Jensen, Adam Iverson, Paulo Rigg

# Desired outcome

$$f(t) \rightarrow \omega(t) \rightarrow u_P(t)$$

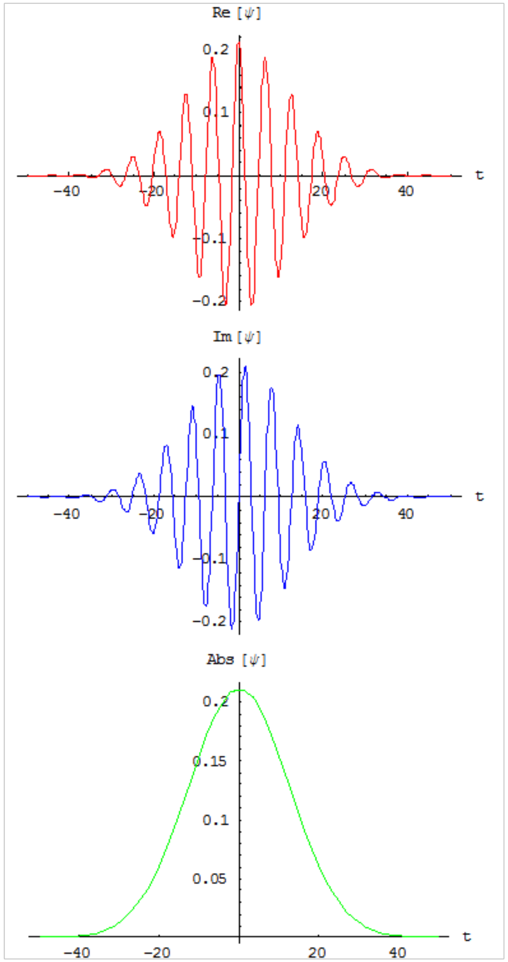


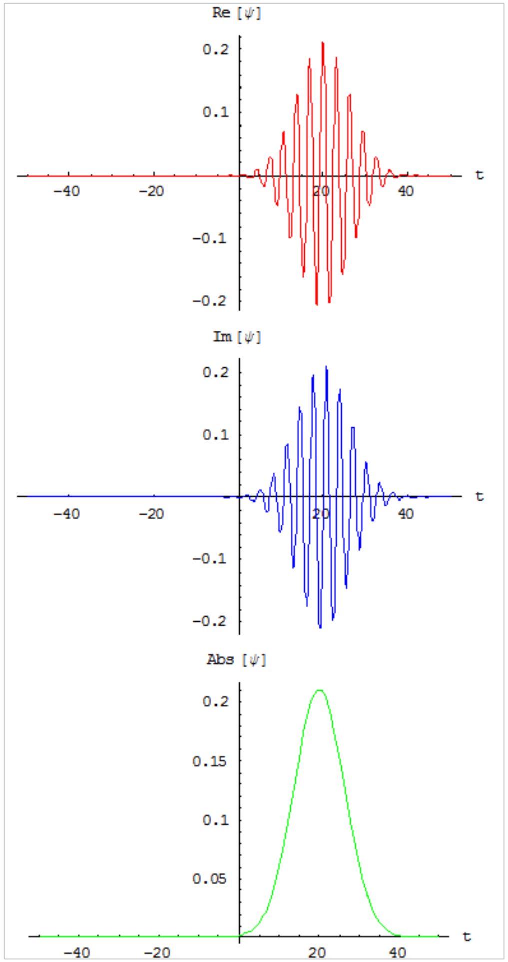

Joint optimization of time and frequency response

# What is a wavelet?

“mother” wavelet




Shifted and scaled
“daughter” wavelet




- Signal represented as a sum of wavelets.
- Many different wavelets with different desirable and undesirable properties.
- Morlet
- Mexican hat
- Daubechies
- Coiflets
- Hermitian
- And many more

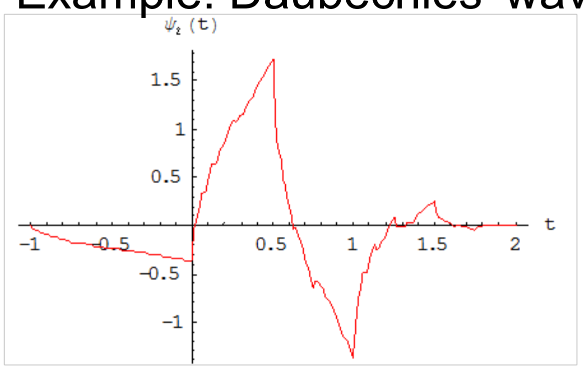
# Wavelet links and references

- http://www.answers.com/topic/wavelet?cat=technology
- http://users.rowan.edu/~polikar/WAVELETS/WTtutorial.html
- http://www.amara.com/current/wavelet.html (general links)
- J. D. Harrop, "Structural properties of Amorphous Materials," PhD Thesis, University of Cambridge, 2004, chapters 2,3 http://www.ffconsultancy.com/free/thesis.html
- http://www.ffconsultancy.com/products/CWT/index.html

# Two kinds of wavelets/wavelet transforms

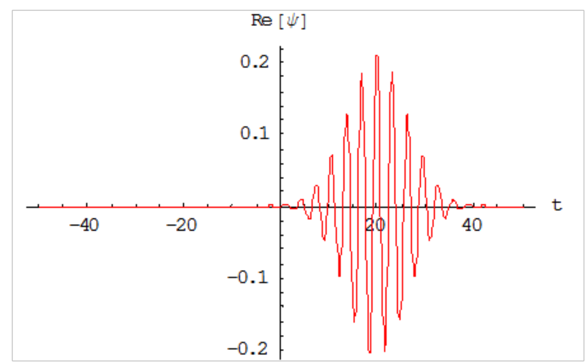
**Discrete Wavelet Transform (DWT)**

- Commonly used for data/image compression
- JPEG 2000
- MP3 2000
- Fast, compact, efficient
- Not typically used for time/frequency analysis
- Example: Daubechies' wavelet




**Continuous Wavelet Transform (CWT)**

- often used for time/frequency analysis
- Computationally intensive, slow ($O(n^2 \ln n)$)
- Example: Morlet wavelet



# The continuous wavelet transform and the short time or windowed Fourier transform

**Windowed Fourier Transform**

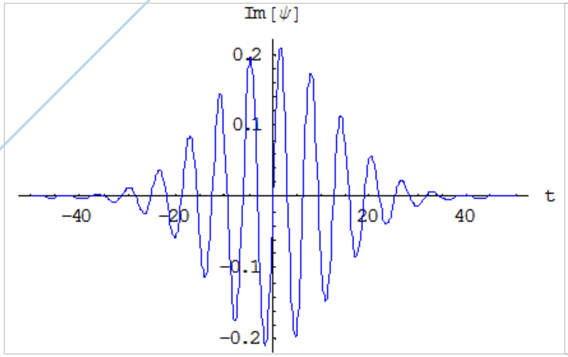
$$F(t,\omega) = (2\pi)^{-\frac{1}{2}} \int_{-\infty}^{\infty} f(t')e^{-i\omega t'} W(t'-t)dt'$$

$$F(t,\omega) = (2\pi)^{-\frac{1}{2}} \int_{-\infty}^{\infty} f(t')e^{-i\omega t'} \exp(-k(t'-t)^2)dt'$$

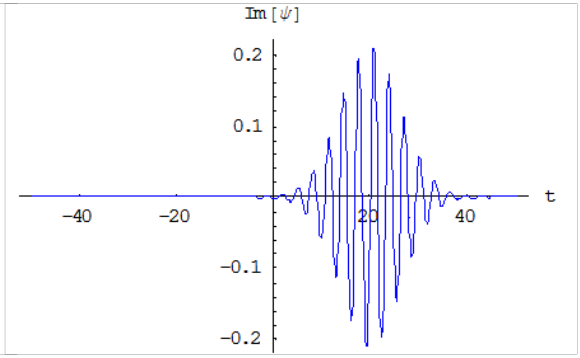
**Continuous Wavelet Transform (CWT)**

$$F_\Psi(t,a) = |a|^{-1} \int_{-\infty}^{\infty} f(t')\Psi^*\left(\frac{t'-t}{a}\right)dt'$$

Good wavelets for time frequency analysis look like a Gaussian multiplied by a complex trigonometric function.
Tuneable.

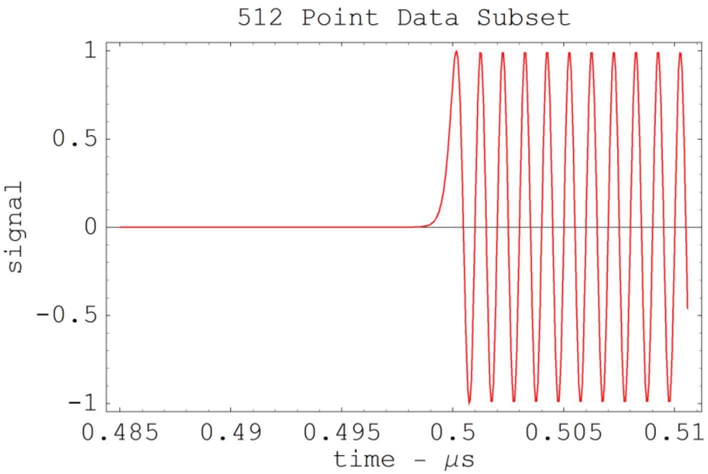


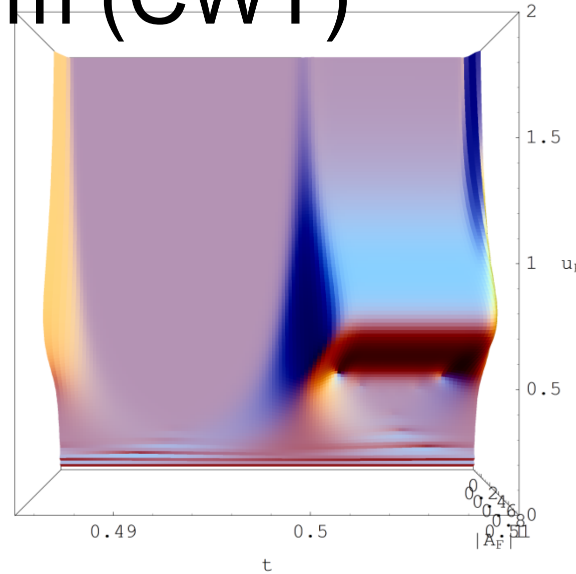
wavelet



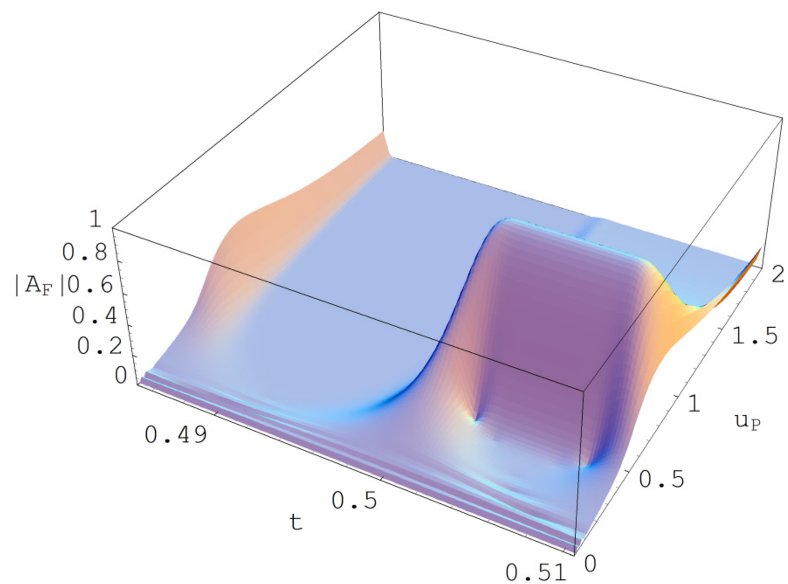
shifted scaled wavelet

# Analysis of signal with a Continuous Wavelet Transform (CWT)

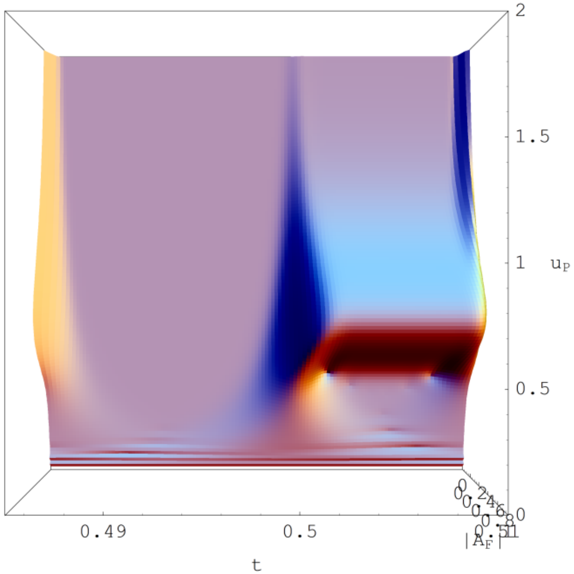


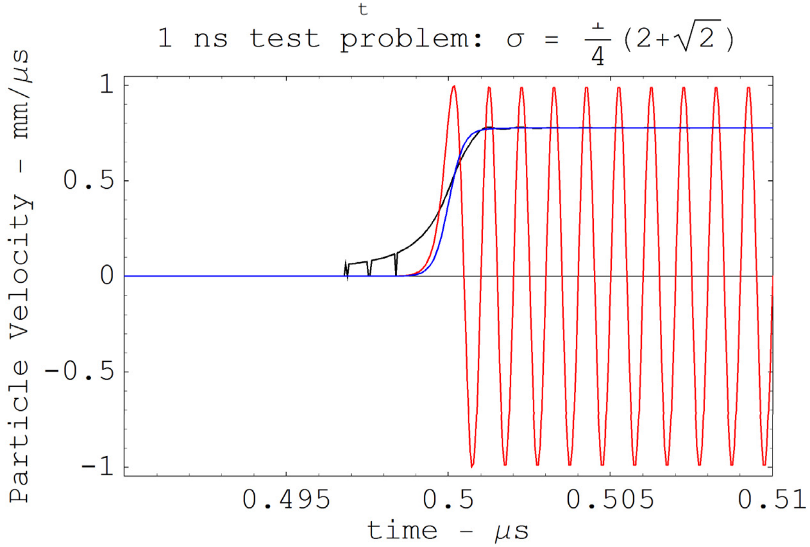




- The original data set and two views of its Continuous Wavelet Transform (CWT).
- Ridge forms where wavelet has same frequency as signal.
- Next step in analysis involves locating the "ridge" or ridges.



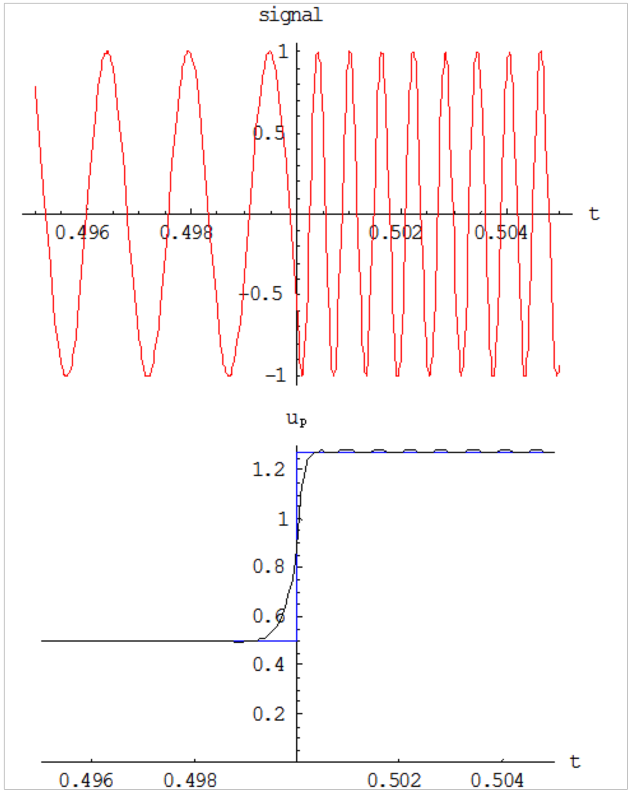
# Particle velocity extracted from transform ridge

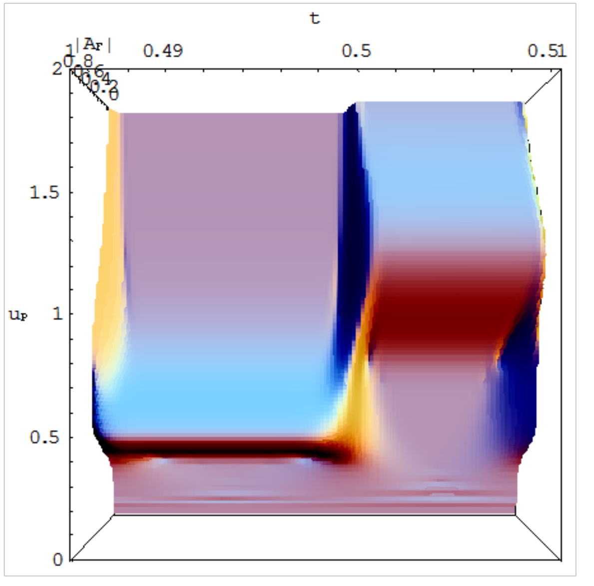



- Red = signal, blue = true velocity, black = velocity from analysis.
- Analysis starts to "lock on" to true velocity after ~ ½ fringe.
- "Precursor" due to "problems" near edge of transform domain. Problems are in both time and frequency (or velocity) directions.
- Are the problems due to the method of solution?

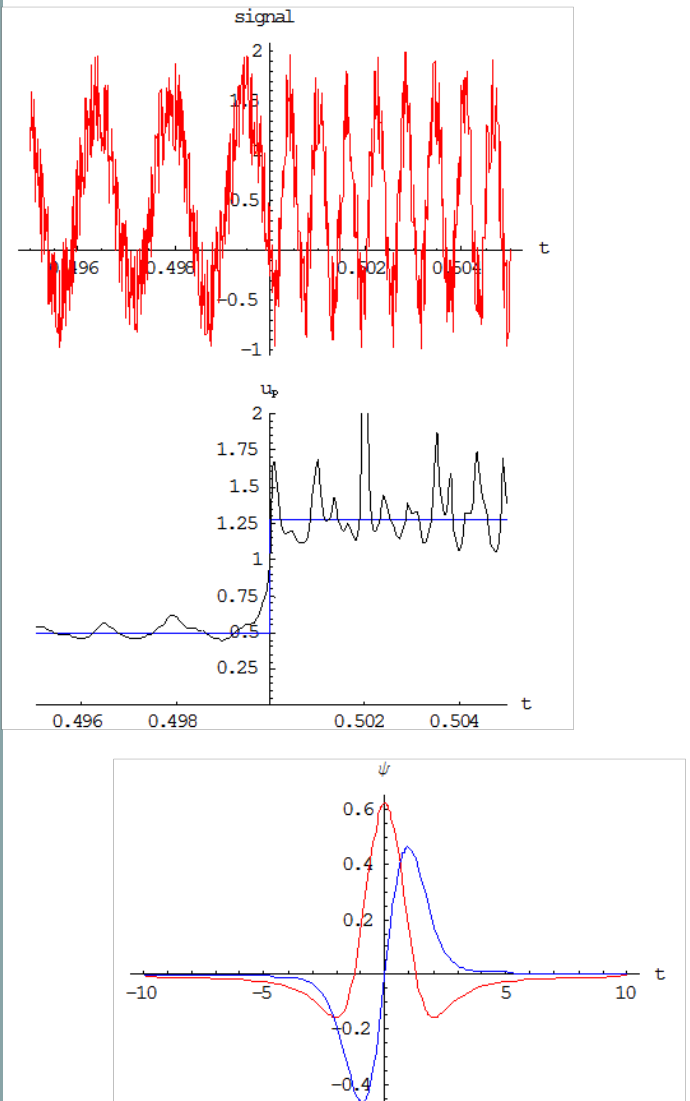
# Velocity step from 0.5 – 1.275 km/s



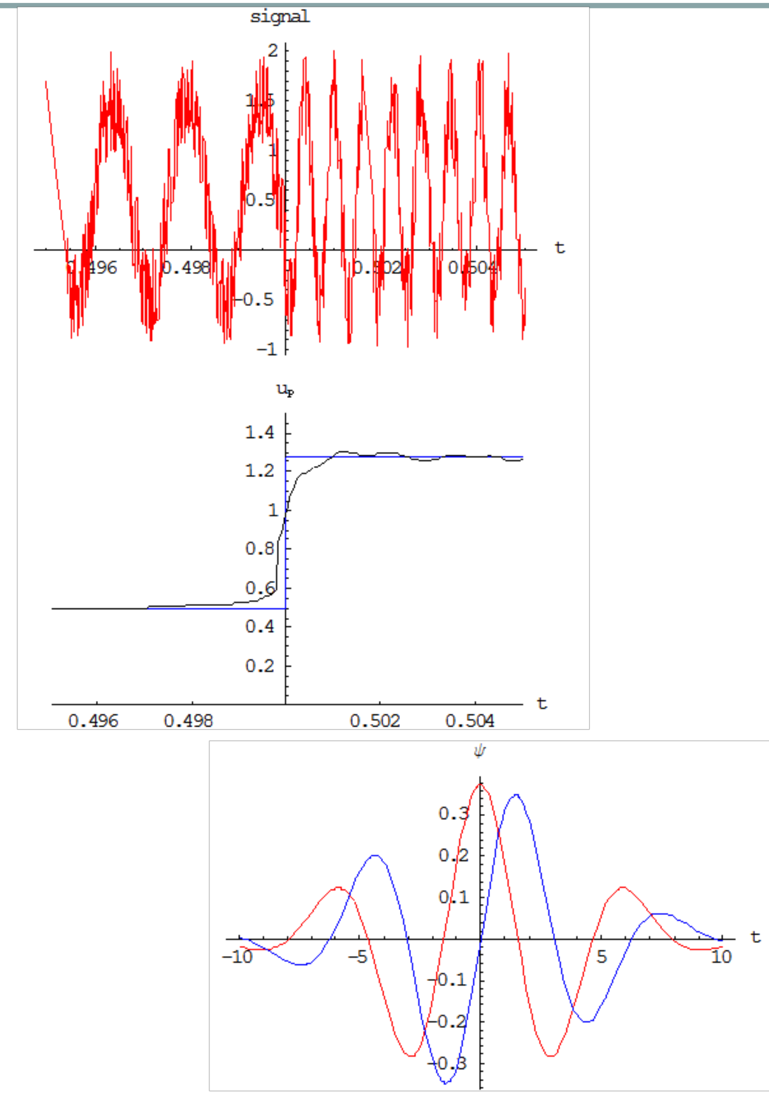




- Red = signal, blue = true velocity, black = waveform from analysis.
- We lock into the right velocity < ½ fringe on either side of the frequency change.
- The fraction of a fringe is the relevant measure of the time response. (1 km/s → 1.3 GHz → 0.8 ns.)

# Velocity step with added noise

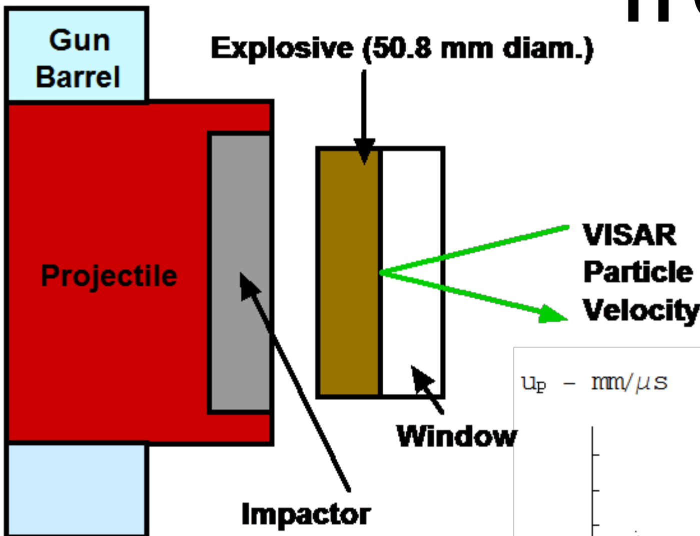


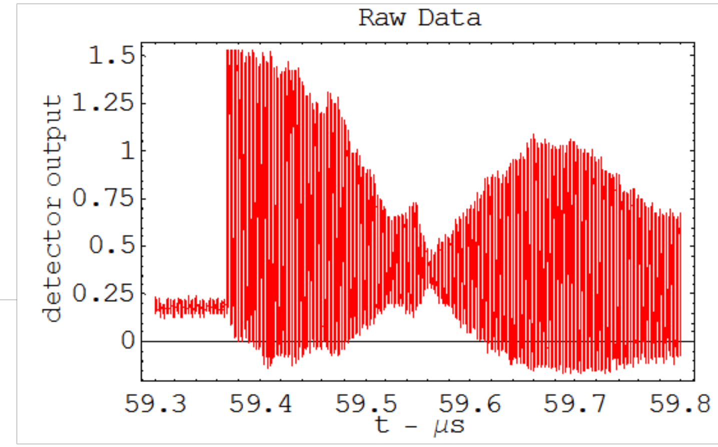

Few oscillations in wavelet

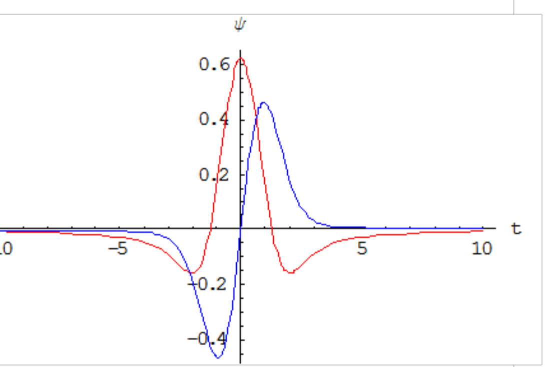



More oscillations in wavelet

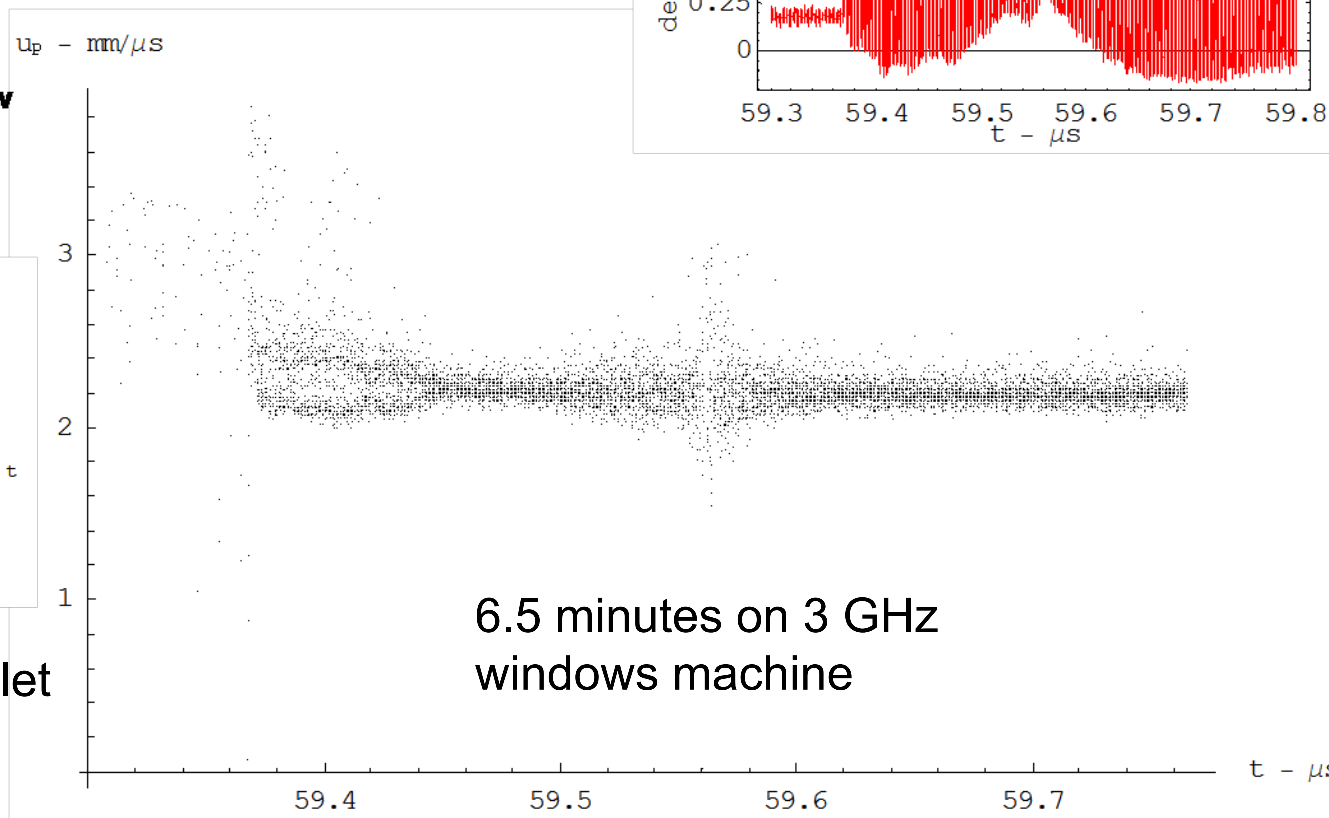
# Overdriven PBX 9502 (data from Jensen)



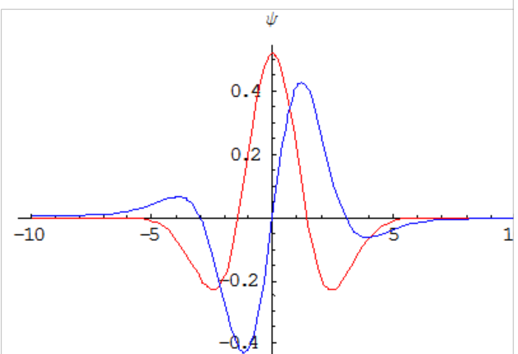




Low oscillation wavelet

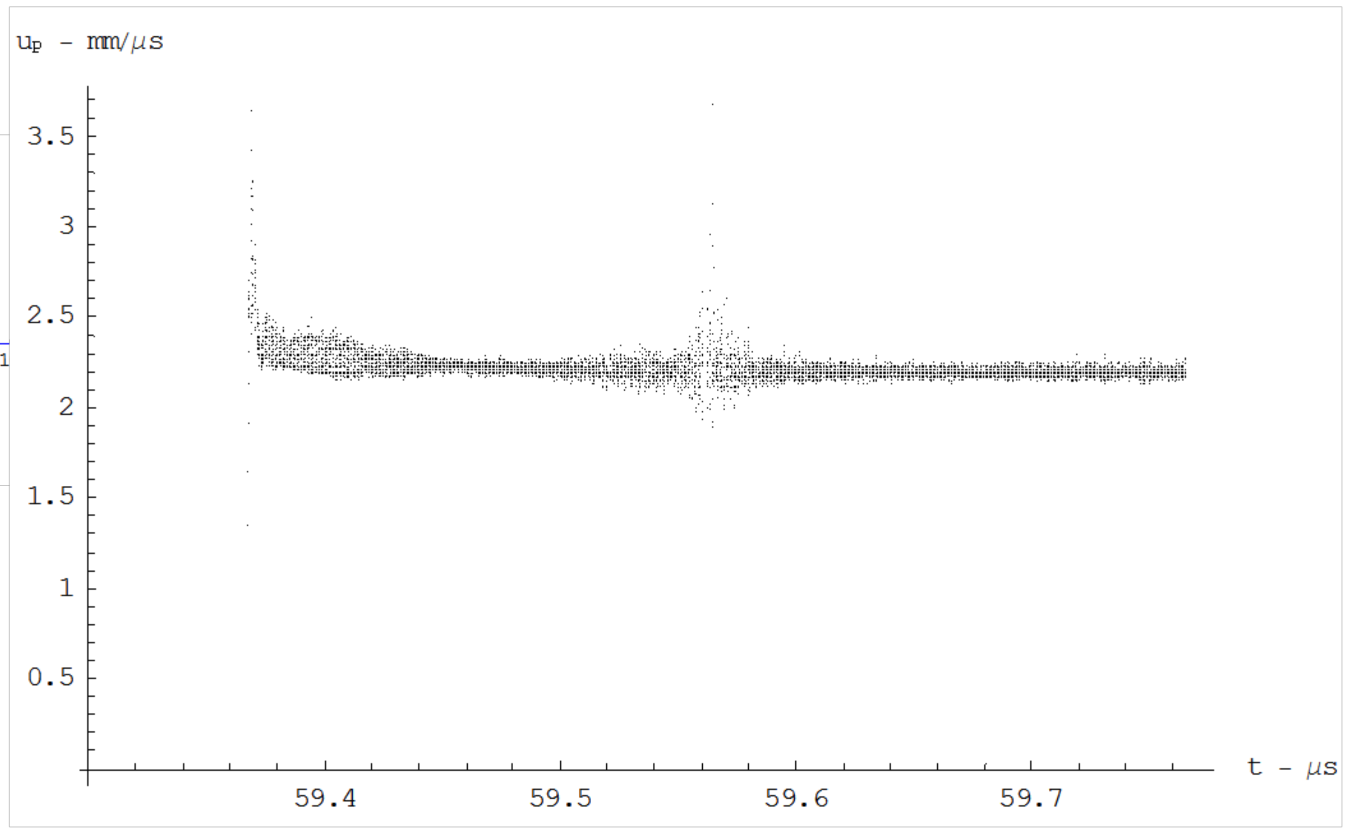



6.5 minutes on 3 GHz windows machine

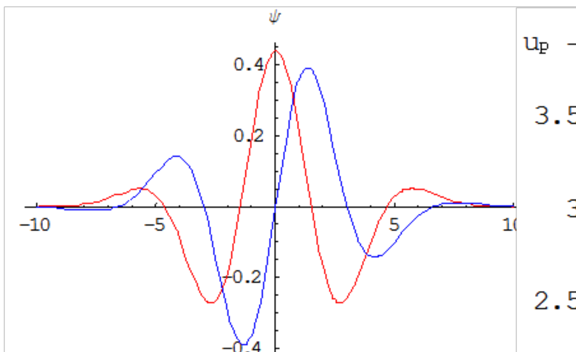
# Jensen data analyzed with more oscillations in wavelet



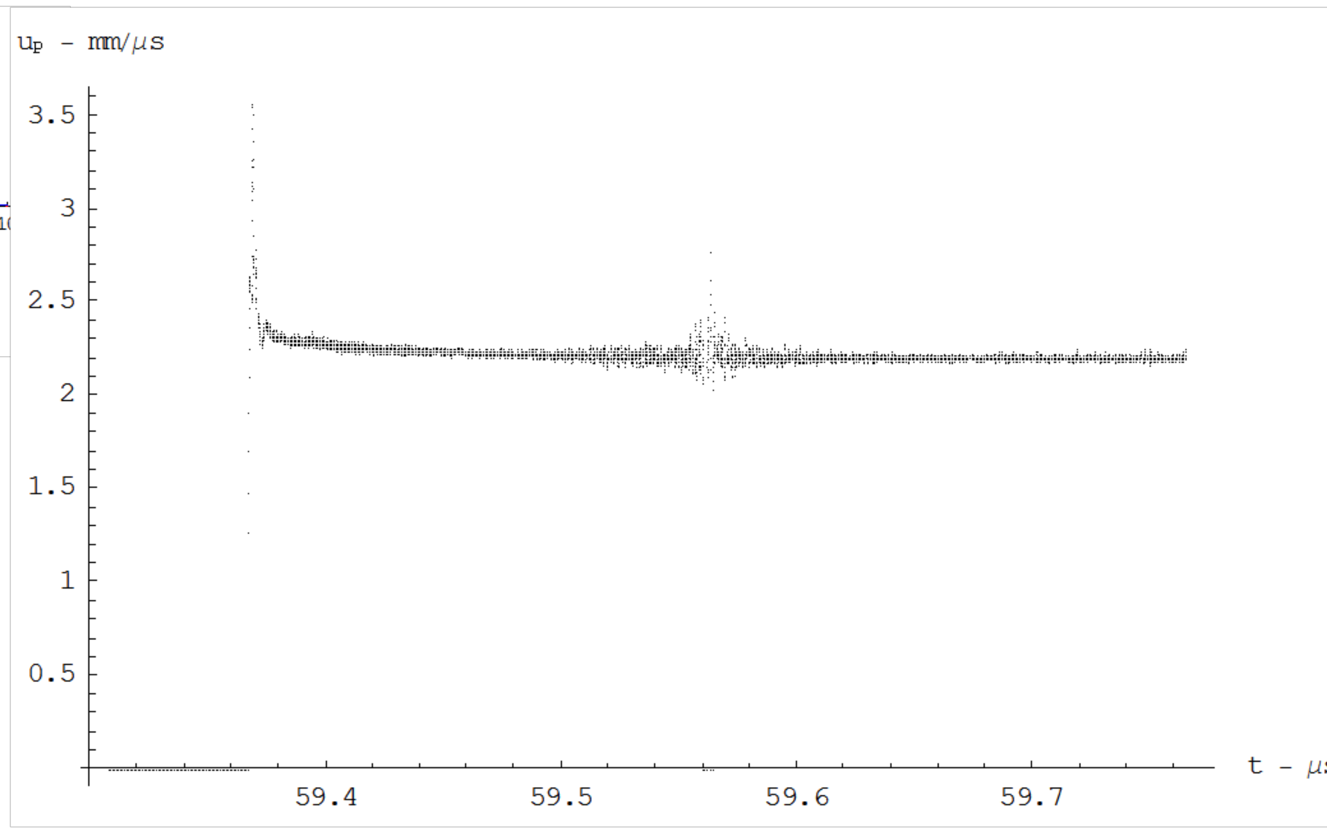

More oscillations



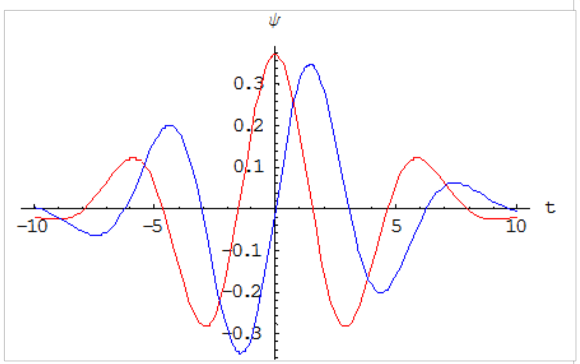
# Overdriven PBX 9502 (data from Brian Jensen)

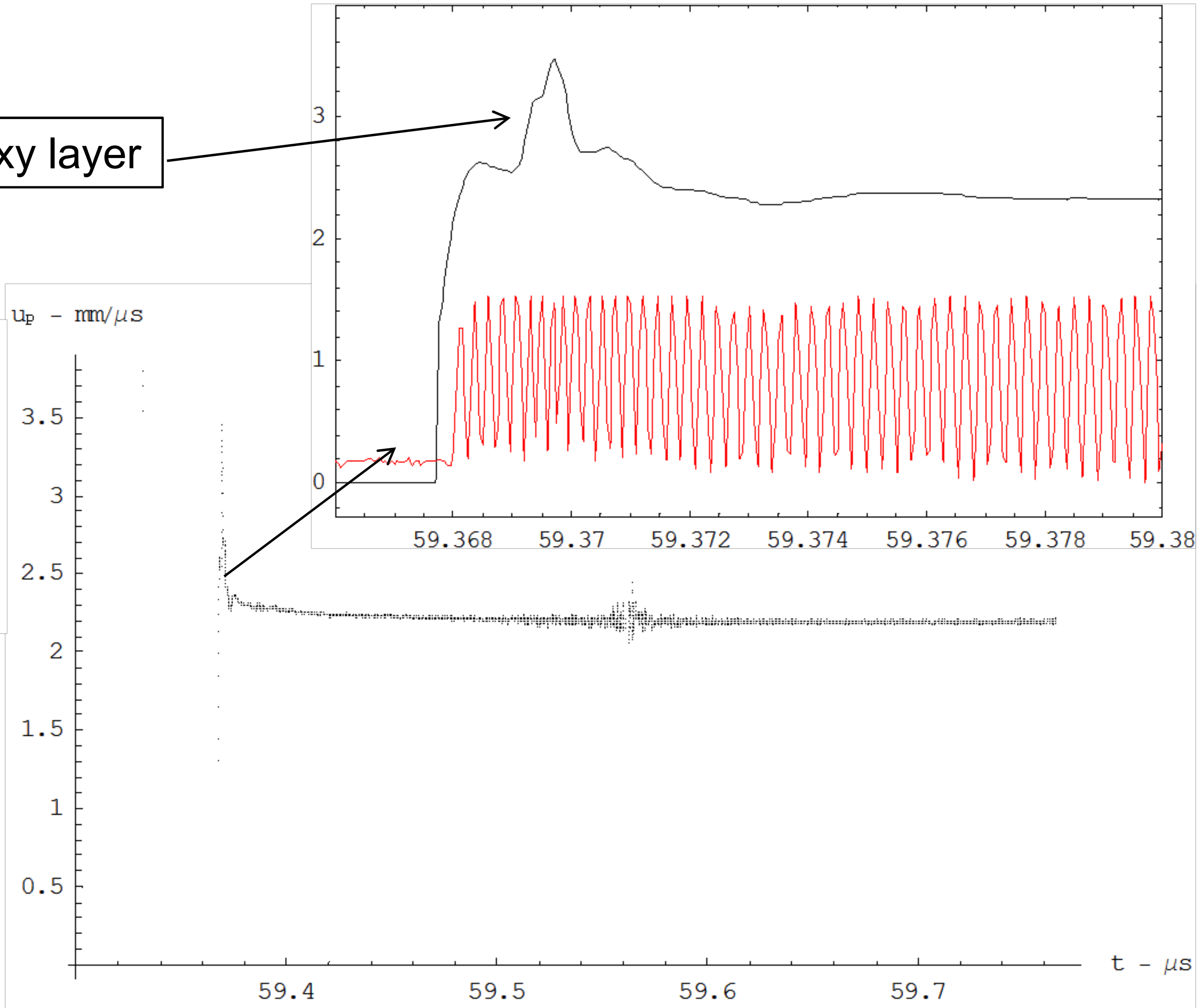



Even more oscillations



# Enough oscillations in wavelet?




still more oscillations

# Summary

- Wavelets are **another** viable technique for analyzing PDV signals.
- Many similarities with Gaussian windowed Fourier Transform.
- User should educate him/herself and use with caution.
- The CWT is computationally intensive and slow, but tractable.
- Time resolution on the order of ½ fringe (or sine wave cycle) can be achieved with perfect data.
- Time resolution drops to 1 – 2 fringes with noisy (typical) data.
- The ramp leading into a steady sine wave can't be properly tracked. Can anything but the Hilbert Transform track these?