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PHOTOMETRIC MICROLENS PARALLAXES WITH THE *SPACE INTERFEROMETRY MISSION*

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ABSTRACT

Astrometric measurements of microlensing events can in principle determine both the "parallax" $\tilde{r}_E$ and the "proper motion" μ of an individual event, which (combined with the Einstein timescale t_E) in turn yield the mass, distance, and transverse velocity of the lens. We show, however, that the parallax measurements are generically several orders of magnitude less precise than the proper-motion measurements. Fortunately, astrometric measurements by the *Space Interferometry Mission* (*SIM*) are simultaneously *photometric* measurements, and since *SIM* will be in solar orbit, these allow *SIM* to be used as a classical (photometric) parallax satellite. We show that *SIM* photometric parallaxes are of precision comparable to that of its astrometric proper-motion measurements. For $I = 15$ bulge stars, complete solutions with $\sim 5\%$ accuracy in mass, distance, and transverse velocity can be obtained from about 5 hr of observation, 100–10,000 times shorter than would be required for a purely astrometric solution of similar precision. Thus, it should be possible to measure directly the mass functions of both the bulge and the inner disk (including both dark and luminous objects) with only a few hundred hours of *SIM* observations.

Subject headings: astrometry — Galaxy: stellar content — gravitational lensing — Magellanic Clouds

1. INTRODUCTION

Boden, Shao, & Van Buren (1998) have shown that it is in principle possible to obtain complete solutions for microlensing events from a series of astrometric measurements using the *Space Interferometry Mission* (*SIM*) or possibly ground-based interferometers. This would be extremely important if practical because two major questions that are difficult to answer on the basis of present-day data could then be easily resolved.

First, after almost a decade of observations, the nature of the events currently being detected toward the Large Magellanic Cloud (LMC) by the MACHO (Alcock et al. 1997b) and EROS (Aubourg et al. 1993) collaborations is a complete mystery. On the one hand, the observed optical depth $\tau \sim 2 \times 10^{-7}$ is an order of magnitude higher than expected from known populations of stars. On the other hand, if the lenses lie in the Galactic halo and so comprise of order half the dark matter, then their masses (inferred from the event timescales and kinematic models of the halo) are of order half a solar mass. Thus, the objects could not be made of hydrogen or they would easily have been discovered from star counts (Alcock et al. 1997b and references therein). Direct measurements of the mass and distance of the lenses would unambiguously resolve this question.

Second, there appears to be a large excess of short-timescale events toward the Galactic bulge relative to what would be expected if bulge stars had a mass function similar to that seen in the solar neighborhood (Han & Gould 1996). The events would be explained easily if the mass function were rising more steeply toward low masses (Zhao, Spergel, & Rich 1995; Han 1997), but recent observations of the bulge by Holtzman et al. (1998) show that the bulge luminosity function is very similar to the local one. By now hundreds of events have been discovered toward the bulge, although only about 50 have been published (Udalski et al. 1994; Alcock et al. 1997a). If individual masses, positions, and velocities of even 10% of these could be measured, our knowledge of the bulge population (both dark and luminous) would be dramatically increased.

In addition, the PLANET (Albrow et al. 1999) and MPS (Rhie et al. 1999) collaborations are currently searching for planetary systems by closely monitoring ongoing microlensing events seen toward the Galactic bulge. Ordinarily, these observations can yield only the planet/star mass ratio and their projected separation in units of the Einstein radius of the lens (Mao & Paczyński 1991; Gould & Loeb 1992). Complete solutions of the event would enable one to translate these quantities into planet masses and physical projected separations.

At present, the only quantity routinely measured for all events is the Einstein timescale, t_E, which is a complicated combination of the physical parameters that one would like to know,

$$t_E = \frac{\theta_E}{\mu}\,, \quad \mu = \frac{v}{D_{ol}}\,, \tag{1}$$

where v is the transverse speed of the lens relative to the observer-source line of sight, μ is the proper motion, and θ_E is the angular Einstein radius,

$$\theta_E = \left(\frac{4GM}{Dc^2}\right)^{1/2}, \quad D \equiv \frac{D_{ol} D_{os}}{D_{ls}}\,. \tag{2}$$

Here M is the mass of the lens, and D_{ol}, D_{os}, and D_{ls} are the distances between the observer, lens, and source. There are numerous ideas on how to get additional information about individual events, but these often require special circumstances. For example, if the source crosses a caustic in the lens geometry, then it is possible to measure the proper motion μ and so, from equation (1), the Einstein radius (Gould 1994a; Nemiroff & Wickramasinghe 1994; Witt & Mao 1994). In fact, a variant of this technique has recently been used to measure the proper motion of a lens seen toward the Small Magellanic Cloud (SMC) and so demonstrate that the lens almost certainly resides in the SMC (Afonso et al. 1998; Albrow et al. 1999; Alcock et al. 1999; Udalski et al. 1998; Rhie et al. 1999). However, such caustic crossing events are rare, and the great majority of them are binaries (and hence may not be representative of the lens population as a whole).

A second type of information can come from parallax measurements. If the event is sufficiently long, then the normal light curve is distorted by the accelerated motion of the Earth about the Sun, allowing one to measure the Einstein radius projected onto the observer plane, $\tilde{r}_E$,

$$\tilde{r}_E = D\theta_E \tag{3}$$

(Gould 1992). Several parallaxes have been measured for bulge events (Alcock et al. 1995; D. Bennett 1998, private communication), but all for events that are substantially longer than typical. It would be possible to measure the parallaxes of microlensing events routinely by launching a satellite into solar orbit (Refsdal 1966; Gould 1995b). The event would have a different time of maximum magnification, t_0, and different impact parameter, β, as seen from the Earth and the satellite. From the differences in these quantities, Δt_0 and $\Delta\beta$, and using the known Earth-satellite separation, $d_{\oplus\text{-}s}$, and known angle γ between the line of sight and the Earth-satellite vector, one could reconstruct both the size of the projected Einstein ring, $\tilde{r}_E$, and the direction of motion, ϕ, relative to the satellite-Earth vector:

$$\tilde{r}_E = \frac{d_{\oplus\text{-}s}|\sin\gamma|}{[(\Delta\beta)^2 + (\Delta t_0/t_E)^2]^{1/2}}\,, \quad \tan\phi = \frac{\Delta\beta}{\Delta t_0}\,. \tag{4}$$

A rather technical but in the present context very important point is that it is significantly easier to measure the difference in times of maximum, Δt_0, than it is to measure the difference in impact parameters, $\Delta\beta$. There are two interrelated reasons for this, which are investigated in detail by Gould (1994b, 1995b), Boutreux & Gould (1996), and Gaudi & Gould (1997). First, the sign of the impact parameter, β, measured by a single observer is intrinsically ambiguous because the light curve contains no information about the side of the lens on which the source passes. Hence, from the two individual impact-parameter measurements, $\beta_\oplus$ and β_s, it is possible to reconstruct four different values of $\Delta\beta = \pm(\beta_\oplus \pm \beta_s)$. Second, one must determine β and t_0 from the light curve simultaneously with three other parameters, t_E, F_0, and F_b, the latter two being the fluxes from the source star and any unlensed background light that is blended into the photometric aperture of the source. While t_0 is virtually uncorrelated with any of the other three parameters, β is highly correlated with all of them, in particular with F_b. Hence β (and so $\Delta\beta$) is more poorly measured than t_0 (and Δt_0). While it is possible to break the fourfold discrete degeneracy, this requires measurement of a higher order effect.

No dedicated parallax satellite is currently planned. However, the *Space Infrared Telescope Facility* (*SIRTF*) could be used to measure parallaxes of at least some events. Because *SIRTF* makes its measurements in passbands that are inaccessible from the ground, the relative blending between the Earth and satellite is completely unconstrained, so measurement of $\Delta\beta$ is not simply difficult, it is virtually impossible. Nevertheless, if Δt_0 is well constrained by Earth-satellite observations, then it is possible to determine $\Delta\beta$ from vigorous ground-based observations (Gould 1999a).

In the best of all possible worlds, one would measure both θ_E and $\tilde{r}_E$. These (together with the routinely measured t_E and the approximately known source distance) would then yield a complete solution for M, D_{ol}, and v (e.g., Gould 1995c). For example,

$$M = \frac{c^2}{4G}\,\theta_E\tilde{r}_E\,. \tag{5}$$

At present, this is possible by ground-based measurements only for certain rare classes of events (Hardy & Walker 1995; Gould & Andronov 1999; Gould 1997). If there were a parallax satellite, then it would also be possible for those rare events which happened to be accessible to proper-motion measurement. However, astrometric microlensing opens the possibility, at least in principle, that such complete measurements might be made for a large unbiased sample of events in the future.

2. ASTROMETRIC MICROLENSING: PROMISE AND LIMITATIONS

As Boden et al. (1998) discuss, astrometric measurements are sensitive to two distinct effects. First, the center of lensed light from the source is displaced from the actual position of the source by

$$\delta\boldsymbol{\theta} = \frac{\boldsymbol{u}_\odot}{u_\odot^2 + 2}\,\theta_E\,, \tag{6}$$

where $\boldsymbol{u}_\odot \equiv \boldsymbol{\mu}(t - t_0)/\theta_E$ is the projected position of the source relative to the lens in units of angular Einstein radius, assuming rectilinear motion as would be observed from the Sun. That is,

$$u_\odot(t) = \left[\frac{(t-t_0)^2}{t_E^2} + \beta^2\right]^{1/2}\,. \tag{7}$$

This deviation traces out an ellipse with semimajor and semiminor axes,

$$\theta_a = \frac{1}{2(\beta^2+2)^{1/2}}\,\theta_E\,, \quad \theta_b = \frac{\beta}{2(\beta^2+2)}\,\theta_E\,. \tag{8}$$

The major axis is aligned with the direction of motion of the lens relative to the source. Hence, by measuring this effect, one can solve for both θ_E and the direction of motion. A measurement of θ_E is often called a "proper-motion" measurement because, from equation (1) it can be combined with the known Einstein timescale to yield the magnitude of the proper motion. However, in the case of astrometric measurements, it also yields the direction, ϕ, and so the full vector proper motion, $\boldsymbol{\mu}$. Note that because the astrometric effect dies off very slowly ($\propto u^{-1}$), stars not associated with the photometric microlensing event can cause significant shifts in the apparent position of the source. However, because this shift remains nearly constant during the event, it does not interfere with the measurement of $\boldsymbol{\mu}$ (Dominik & Sahu 1999).

The second effect is a parallax deviation caused by motion of the Earth about the Sun. The exact formula for the combined parallax and proper-motion effect can be found by substituting

$$\boldsymbol{u}_\oplus = \boldsymbol{u}_\odot - \frac{\hat{\boldsymbol{n}}\times\hat{\boldsymbol{n}}\times\boldsymbol{a}}{\tilde{r}_E} \tag{9}$$

in equation (6). Here $\hat{\boldsymbol{n}}$ is the unit vector in the direction of the source, and $\boldsymbol{a}$ is the position vector of the Earth relative to the Sun. Thus, the magnitude of the perturbative term is

$\sim$AU$/\tilde{r}_E$, which might be $\sim$10%–30% for typical lensing events. This would seem to imply that one could determine the parallax ($\tilde{r}_E$) about 10%–30% as accurately as the proper motion (θ_E) for the same set of measurements.

Unfortunately, the situation is not so favorable. The perturbation in equation (9) is not directly observable because there are no comparison observations from the Sun. Consider the limit $t_E \ll \mathrm{yr}/2\pi$, which is typical for events seen toward the Galactic bulge. In this case, the Earth's velocity would barely change during the event or even for the first few t_E after it. One would then see the same ellipse as described by equation (8), but with a β different from the one that would have been seen from the Sun. Ninety days after the event, the direction of apparent source motion would have changed by an angle $v_\oplus/\tilde{v}$, where $\tilde{v} \equiv \tilde{r}_E/t_E$ is the speed of the lens projected onto the observer plane, and $v_\oplus = 30$ km s^{-1} is the speed of the Earth. According to equation (6), this would introduce an astrometric deviation of order $(v_\oplus/\tilde{v})(t_E/90\ \mathrm{days})\theta_E$. Since θ_E is essentially determined from the major axis of the ellipse (eq. [8]), which is approximately $2^{-1/2}\theta_E$, the relative size of the astrometric parallax and proper-motion measurement errors is roughly given by

$$\Gamma_{\mathrm{ast/ast}} \equiv \frac{\sigma_{\tilde{r}_E,\mathrm{ast}}/\tilde{r}_E}{\sigma_{\theta_E,\mathrm{ast}}/\theta_E} \sim \frac{\tilde{v}}{v_\oplus}\,\frac{60\ \mathrm{days}}{t_E}\,. \tag{10}$$

In fact, equation (10) is too optimistic in that it implicitly assumes that the direction of the major axis of the ellipse can be determined with infinite precision. As we will show in § 5, this is very far from the case. Hence, the true ratio of errors is generally larger than that implied by equation (10). For typical bulge-bulge lensing events, $\tilde{v} \sim 800$ km s^{-1}. For typical halo events seen toward the LMC, $\tilde{v} \sim 300$ km s^{-1}. For lenses in the LMC, $\tilde{v}$ is a factor of 3–10 higher still.

The above analysis implies that astrometric microlens parallax measurements are several orders of magnitude less accurate than proper-motion measurements. This would not present much of a problem if very accurate proper-motion measurements could be made with a modest amount of observing time. However, as we will show in § 5, even for bright ($I \sim 15$) sources seen toward the bulge, proper-motion measurements accurate to 5% require about 5 hr of observations. Very few events seen toward the LMC are brighter than $V \sim 20$, and therefore $\sim$40 times more observing time is needed to achieve the same precision. Hence, this analysis appears to imply that no more than a few accurate microlens parallaxes could be obtained in any reasonable observing program.

3. PHOTOMETRIC MICROLENSING PARALLAXES WITH *SIM*

SIM is not designed to do photometry, and it would seem completely hairbrained to waste this precision astrometric instrument on measurements that could be done more efficiently from the ground using telescopes with collecting areas that are several orders of magnitude larger. Nevertheless, two unrelated factors combine to make *SIM* the ideal device to measure microlens parallaxes photometrically (rather than astrometrically).

3.1. *Photometry with SIM*

First, *SIM* works by *counting photons* as a function of position in the interference pattern in order to find the centroid of the central fringe. The photons are distributed in this fringe as $NF(\theta)d\theta$, where N is the total number of photons in the central fringe,

$$F(\theta) = \frac{1}{\pi\theta_f}\cos^2\left(\frac{\theta}{2\theta_f}\right), \quad \theta_f \equiv \frac{\lambda}{2\pi d}\,, \tag{11}$$

d is the distance between the mirrors, and λ is the wavelength of the light. The astrometric precision is given by (e.g., Gould 1995a)

$$\sigma_\theta = N^{-1/2}\left[\int d\theta\, F(\theta)\left(\frac{d\ln F}{d\theta}\right)^2\right]^{-1/2} = N^{-1/2}\theta_f\,, \tag{12}$$

and hence the fractional photometric precision ($\sigma_{\mathrm{ph}} = N^{-1/2}$) is related to the astrometric precision by

$$\sigma_{\mathrm{ph}} = \frac{\sigma_\theta}{\theta_f}\,. \tag{13}$$

3.2. *One-dimensional Photometric Parallaxes*

Second, as discussed in § 1, the real problem in obtaining microlens parallaxes photometrically is that the microlens parallaxes are inherently two-dimensional. In effect, by measuring Δt_0, one determines $\cos\phi/\tilde{r}_E$, and by measuring $\Delta\beta$, one determines $\sin\phi/\tilde{r}_E$, where ϕ is the angle of source-lens relative motion with respect to the direction of the *SIM*-Earth axis at the moment when the event is a maximum as seen from the *SIM*-Earth midpoint. It is only by measuring *both* of these quantities that one can determine $\tilde{r}_E$. Since $\Delta\beta$ is difficult to measure, obtaining a precise $\tilde{r}_E$ is also difficult.

As discussed in § 1, it is possible in principle to break the degeneracy in $\Delta\beta$ photometrically if the photometry is good enough. We will show in § 5 that *SIM* photometry is sufficiently precise for this task provided that the observations are carefully planned. However, it is also the case that *SIM* astrometric measurements *by themselves* often determine ϕ (from the orientation of the ellipse) with sufficient precision to break the degeneracy in $\Delta\beta$. In these cases, $\tilde{r}_E$ can be determined from a measurement of Δt_0 alone. In this paper we will consider both methods of breaking the degeneracy, but in the remainder of this section we will assume that the degeneracy is broken astrometrically. This will allow us to estimate $\Gamma_{\mathrm{ph/ast}}$, the relative precision of *SIM* photometric parallax measurements to *SIM* astrometric proper-motion measurements. (Recall from the discussion following eq. [4] that blending does not significantly affect the measurement of Δt_0, and therefore it will not be considered in this section. We will give a thorough discussion of blending in § 4.)

As currently designed, *SIM* will fly in a *SIRTF*-like orbit, drifting away from the Earth at about 0.1 AU yr^{-1}. Let the distance at the time of the observations be $d_{\oplus\text{-}s}$. Then $\tilde{r}_E$ can be determined from the measured Δt_0 (and the known value of ϕ) by

$$\tilde{r}_E = d_{\oplus\text{-}s}\,\frac{t_E}{|\Delta t_0|}\,|\cos\phi|\,. \tag{14}$$

Gould (1999a) analyzed how to optimize measurements of Δt_0 when he investigated microlens parallaxes with *SIRTF*. For photon-limited photometry, one should concentrate the measurements near times $t_\pm$ before and after the peak, where $t_\pm = t_0 \pm (5/3)^{1/2}\beta t_E$. The error in $t_{0,s}$ is then given approximately by

$$\sigma_{t_{0,s}} \sim n^{-1/2}\sigma_{\rm ph}\,\frac{\beta}{0.5}\,t_{\rm E} \quad (\beta \lesssim 0.5)\,, \tag{15}$$

where n is the total number of measurements in these two regions. We will assume that the ground-based measurements to determine $t_{0,\oplus}$ have precision similar to that of the *SIM* measurements. Equation (14) then implies that the fractional error in $\tilde{r}_{\rm E}$ is given by

$$\frac{\sigma_{\tilde{r}_{\rm E}}}{\tilde{r}_{\rm E}} = 2^{1/2}\,\frac{\sigma_{t_{0,s}}}{|\Delta t_0|} = 2^{1/2}\,\frac{\tilde{v}\sigma_{t_{0,s}}}{d_{\oplus\text{-}s}}\,|\sec\phi|\,. \tag{16}$$

Then, assuming that a similar number of measurements are used to determine the semimajor axis of the ellipse, θ_a, and to determine Δt_0 (they are somewhat the same measurements), the ratio of the *photometric* precision of the parallax to the *astrometric* precision of the proper motion is

$$\Gamma_{\rm ph/ast} = \frac{\sigma_{\tilde{r}_{\rm E}}/\tilde{r}_{\rm E}}{\sigma_{\theta_a}/\theta_a} \sim \frac{\tilde{r}_{\rm E}\,\sigma_{\rm ph}(\beta/0.3)\,|\sec\phi|/d_{\oplus\text{-}s}}{\sigma_\theta/(\theta_{\rm E}/3)}\,, \tag{17}$$

where we have approximated equation (8) as $\theta_a = \theta_{\rm E}/3$. Using equations (5) and (13), this can be rewritten as

$$\Gamma_{\rm ph/ast} \sim \beta\,\frac{4GM}{c^2 d_{\oplus\text{-}s}\,\theta_f}\,|\sec\phi|$$
$$\sim \frac{\beta}{0.25}\,\frac{M}{0.3\,M_\odot}\left(\frac{d_{\oplus\text{-}s}}{0.2\ {\rm AU}}\right)^{-1}|\sec\phi|\,, \tag{18}$$

where we have adopted $\theta_f = 2.5$ mas, which is appropriate if the flux-weighted harmonic mean wavelength of the source is 0.8 μm, and the mirrors are separated by 10 m. Equation (18) implies that for typical lenses, the photometric parallax will be of comparable precision to the astrometric proper motion, in sharp contrast to the large ratio for astrometric parallaxes found in equation (10).

In fact, the $|\sec\phi|$ dependence in equation (17) is too pessimistic because we have ignored all photometric information about $\Delta\beta$. We show in § 5 that except for the case $\cos\phi \simeq 0$ (where the discrete degeneracy becomes astrometrically incorrigible) it is possible essentially to eliminate the $|\sec\phi|$ term in equation (17) using a combination of astrometric and photometric data.

4. SIMULATIONS

The estimates given in the previous two sections are useful because they elucidate the relation between the physics of the event and the measurement process, on the one hand, and the errors in the microlensing parameters, on the other. By the same token, however, they cannot capture the full range of experimental conditions, and so are necessarily approximate. The actual errors for any given event will depend both on the precise event parameters and on the observational strategy. While a full investigation of the best observational strategy lies well beyond the scope of the present study, it is important to make a rigorous calculation of the statistical errors for some representative examples in order to obtain more precise estimates and to investigate more subtle effects that are not captured by the rough analysis given above.

For this purpose, we consider a set of somewhat idealized observations. First, we assume that the principal measurements are carried out at uniform time intervals that are short compared to $t_{\rm E}$, beginning when the magnification first reaches $A = 1.5$ and ending at a time that will be determined below from signal-to-noise ratio (S/N) considerations. The assumption of uniform observations is quite reasonable for observations toward the LMC (near the ecliptic pole) but is obviously not really possible toward the bulge (near the ecliptic). For the bulge, we therefore assume that the measurements are interrupted when the bulge is within 60° of the Sun. This measurement strategy can actually be very far from ideal, and we modify it somewhat in § 4.2 below. The assumption that events can be *recognized* at $A = 1.5$ is reasonable, but whether observations can begin as soon as the events are recognized requires additional discussion. *SIM* design characteristics are not yet fixed, but A. Boden (1999, private communication) has provided us with the following summary of the current status based on discussions with the *SIM* Deputy Project Scientist and the *SIM* Mission Planning Manager. The current "requirement" for target-of-opportunity response is 4 days, with a "goal" of 2 days, where a "requirement" is what the project is using as a basis for planning and a "goal" is what the project will attempt to support if resources are available. The mechanics of operating the spacecraft limit the response time absolutely to a minimum of about 16 hr, but to achieve this much faster response would be costly in dollars. Thus, the actual response time of *SIM* will be set by balancing the scientific returns against costs that ultimately limit other capabilities of the satellite. For simplicity, we here assume that observations can always begin at $A = 1.5$. Clearly, the finite response time will degrade this to some degree, particularly for the shortest events. Since the experiment being proposed in this paper will probably stress the target-of-opportunity capability of *SIM* at least as much as any other, a more detailed study of the effects of the delay should be made in the near future. However, this is beyond the scope of the present paper.

Second, we assume that the exposure times are of equal duration, so that the S/N is better near the peak of the event. Third, we assume $\theta_f = 2.5$ mas and $d_{\oplus\text{-}s} = 0.2$ AU, although the first will clearly vary from star to star, and the second will change during the course of the mission. For the LMC (at the ecliptic pole), the time of year at which the event is discovered plays no role, but for the bulge it does. We will assume that the bulge field lies at $B = -6°$ from the ecliptic, close to the (northern) winter solstice. The Earth-satellite separation projected onto the sky is therefore at a maximum at the summer solstice and is $d_{\oplus\text{-}s}(1 - \cos^2\psi\cos^2 B)^{1/2} \sim d_{\oplus\text{-}s}|\sin\psi|$ at other times of the year, where ψ is the phase of the Earth's orbit relative to the autumnal equinox. As discussed in detail by Gaudi & Gould (1997), the orbital phase ψ has two conflicting effects. First, when $\cos\psi \sim 0$ (near the summer solstice), the *SIM*-Earth projected separation is at a maximum, and hence the measurement errors of Δt_0 and $\Delta\beta$ are reduced to a minimum. On the other hand, the relative velocity of *SIM* and the Earth projected onto the plane of the sky is at a minimum, and breaking the degeneracy in $\Delta\beta$ depends critically on this relative velocity. Hence, it is most difficult to break the degeneracy at the summer solstice. As the phase moves away from the summer solstice, the projected separation slowly declines (making the measurements of Δt_0 and $\Delta\beta$ less accurate), but the projected velocity difference rapidly increases (allowing more secure degeneracy breaking). If one relies on photometry to break the degeneracy, the optimal events are those that peak about 45 days from the summer solstice (see, e.g., Fig. 6 of Gaudi & Gould 1997). On the other hand, if it is possible to break the degeneracy astrometrically, then events that peak at the solstice are optimal, since the measurement errors are reduced by $\sim 2^{-1/2}$. In this paper, we will simulate primarily events peaking at $\psi = 225°$, i.e., about May 7. However, we will also discuss events that peak at the summer solstice ($\psi = 270°$).

Next, we will assume for definiteness that the ground-based photometric observations have the same precision as the *SIM* observations. Finally, we will ignore blending in the *SIM* measurements, except blending by the lens. Blending will have an important impact on the overall precision, and hence on the strategy for *SIM* measurements, but, as we show below, it will not affect the main conclusions of this paper, which concern the relative precision of *SIM* astrometric and photometric microlens parallaxes. This is especially true toward the bulge, which will be the main focus of analysis. A proper treatment of blending would therefore make the paper substantially more complex without clarifying any of the central points. Hence, we defer consideration of this important effect to a later paper on observational strategy.

Why can blending be ignored to first order in this analysis? First, Han & Kim (1999) have shown that all potential blends lying more than 10 mas from the source can be eliminated by the *SIM* observations themselves. Since the density of field stars having even a modest fraction of the source flux is much less than 10^4 arcsec^{-2}, this essentially eliminates all blends not directly associated with the event, namely, the lens itself, binary companions to the lens, and binary companions to the source. For bulge events, 10 mas corresponds to 80 AU, so a substantial fraction of binary companions are also eliminated. Second, to minimize observation time, *SIM* observations must be almost entirely restricted to very bright stars (relative to other stars in the field). This means clump giants toward the bulge and either clump giants or early main-sequence stars toward the LMC. The chance that a bulge clump star has a companion within 80 AU and with more than a few percent of its own flux is small because their progenitors are about 50 times fainter than the stars themselves. The primary effect of a few percent blend would be to change the shape and orientation of the proper-motion ellipse and (assuming the shape change went undetected) to therefore change the inferred direction of the lens-source relative proper motion, also by a few percent. This would in turn affect the parallax inferred from Δt_0, which depends on this direction through the angle ϕ. See equation (14). However, this effect on the parallax measurement will also be a few percent. As we will show in § 5, it is quite possible to achieve accuracies of a few percent for bulge events, so a careful investigation of the effect of blending on parallax and proper-motion measurements should be undertaken as part of a more thorough analysis of the problem. Unfortunately, a proper analysis of blending from binary companions to the source requires simulated fits to the entire diffraction pattern, not just the centroid, and so is substantially more involved than the present study. By contrast, low-level blending by the lens can be treated within the framework of the centroid analysis given here, and we therefore include it.

The situation is more complicated toward the LMC because the chance that an early main-sequence star has a companion of comparable brightness is larger, probably a few tens of percent. Even clump stars have brighter companions toward the LMC than toward the bulge because they are younger and so have brighter progenitors. Also, the 10 mas limit on detecting blends directly (Han & Kim 1999) corresponds to 500 AU toward the LMC compared to 80 AU toward the bulge. Nevertheless, even toward the LMC, the majority of sources will not have companions with more than 10% of the source flux, and hence even here it is appropriate to ignore blending by companions in a first treatment.

Toward both the bulge and the LMC, it is very unlikely that the lens itself will contribute more than a small fraction of the source light if the source is bright. We therefore allow for lens blending in our simulated fits but assume that the actual blending is very small.

Note that we will *not* ignore blending in the ground-based photometric observations, since there is no way to eliminate field star blends for the ground-based observations.

4.1. *Parameterization*

We will simulate simultaneous observations from *SIM* and from the ground. There will be four measured quantities: (1) G^1, the flux observed from the ground, (2) G^2, the flux observed from *SIM*, (3) G^3, the x astrometric position, and (4) G^4, the y astrometric position. These give rise to four observational equations:

$$G^1(t) = F_s^1\,A(u_\oplus) + F_b^1\,, \quad G^2(t) = F_s^2\,A(u_{SIM}) + F_b^2 \tag{19}$$

and

$$[G^3(t), G^4(t)] = \frac{\boldsymbol{u}_{SIM}}{u_{SIM}^2 + 2}\,\theta_{\rm E} - \boldsymbol{\alpha}_{SIM}\,\pi_s + \boldsymbol{\mu}_s t + \boldsymbol{\theta}_0 - \boldsymbol{\theta}_b\,, \tag{20}$$

$$\boldsymbol{\theta}_b \equiv \frac{F_b^2\,\theta_{\rm E}}{F_s^2\,A(u_{SIM}) + F_b^2}\left(\frac{u_{SIM}^2 + 3}{u_{SIM}^2 + 2}\,\boldsymbol{u}_{SIM} + \kappa\boldsymbol{\alpha}_{SIM}\right), \tag{21}$$

where $A(u) = (u^2 + 2)/[u(u^2 + 4)^{1/2}]$ is the magnification, $\boldsymbol{\mu}_s$ is the absolute proper motion of the source, $\boldsymbol{\theta}_0$ is the true position of the source at time $t = 0$,

$$\boldsymbol{\alpha} = -\frac{\hat{\boldsymbol{n}} \times \hat{\boldsymbol{n}} \times \boldsymbol{a}}{\rm AU}\,, \tag{22}$$

π_s is the parallax of the source, and $\boldsymbol{u}_{SIM}$ and $\boldsymbol{u}_\oplus$ are defined similarly to equation (9), i.e.,

$$\boldsymbol{u}_{SIM} = \boldsymbol{u}_\odot + \kappa\boldsymbol{\alpha}_{SIM}\,, \quad \boldsymbol{u}_\oplus = \boldsymbol{u}_\odot + \kappa\boldsymbol{\alpha}_\oplus\,, \quad \kappa \equiv \frac{\rm AU}{\tilde{r}_{\rm E}}\,, \tag{23}$$

with

$$\boldsymbol{u}_\odot \equiv (\tau_\odot \cos\phi - \beta_\odot \sin\phi,\, \tau_\odot \sin\phi + \beta_\odot \cos\phi)\,,$$

$$\tau_\odot \equiv \frac{t - t_{0,\odot}}{t_{{\rm E},\odot}}\,. \tag{24}$$

The terms in equation (20) can be understood as follows. The first term is the ellipse characterized by equation (8), modified by the motion of the Earth (i.e., $\boldsymbol{u}_\odot \rightarrow \boldsymbol{u}_{SIM}$). The second term is the parallactic motion of the source. The third and fourth terms represent the ordinary proper motion and position of the source. The last term is the perturbation due to the luminosity of the lens, which is written out explicitly in (21). Here the first term is the difference between the "ellipse" in equation (20) and the relative lens-source position ($-\boldsymbol{u}_{SIM}\,\theta_\mathrm{E}$), while the second is the relative parallax of the lens and the source.

There is a total of 15 parameters: $t_{0,\odot}$, $\beta_\odot$, and $t_{\mathrm{E},\odot}$ are the standard event parameters as the event would be seen from the Sun, ϕ is the direction of source motion relative to the lens with respect to the *SIM*-Earth direction, θ_E is the Einstein radius, κ is the inverse projected Einstein radius (normalized in AU), π_s is the parallax of the source, $\boldsymbol{\mu}_s$ is the proper motion of the source, $\boldsymbol{\theta}_0$ is its position at $t=0$, F_s^1 and F_s^2 are the source fluxes as received by the Earth and satellite observatories, and F_b^1 and F_b^2 are the background fluxes.

To determine the uncertainties in these parameters, we evaluate the covariance matrix c_{ij} (e.g., Gould & Welch 1996)

$$\boldsymbol{c} = \boldsymbol{b}^{-1}\,, \quad b_{ij} = \sum_{l=1}^{4}\sum_{k=1}^{N} \sigma_{kl}^{-2}\,\frac{\partial G^l(t_k)}{\partial a_i}\,\frac{\partial G^l(t_k)}{\partial a_j}\,, \tag{25}$$

where $a_1, \ldots, a_{15}$ are the 15 parameters, t_k are the times of the observations, and σ_{kl} is the error in the measurement of G^l at time t_k. We enforce the condition of weak blending by setting $F_b^1 = F_b^2 = 0$ *after* taking the derivatives in equation (25).

4.2. *Modification of Observing Strategy*

The full problem of optimization of *SIM* microlensing observations lies outside the scope of this paper, but it is straightforward to determine the optimum duration of observations once the (arbitrary) strategy of uniform observations has been adopted: one changes the interval over which the observations are carried out while holding the total observing time fixed, and inspects the resulting errors. We carry out this exercise and find that the optimal duration to determine $\tilde{r}_\mathrm{E}$ is short, typically a few tens of days for various combinations of parameters, while the optimal duration to determine θ_E is well over 100 days. The reason for this is clear. The measurement of $\tilde{r}_\mathrm{E}$ is determined primarily from photometry, and the photometric microlensing event is essentially over after $2t_\mathrm{E}$. On the other hand, θ_E is determined from the astrometric event, which lasts many times t_E. Only after the astrometric event is essentially over is it possible to determine $\boldsymbol{\mu}_s$ and so remove the correlation between this parameter and θ_E.

We address this inconsistency of timescales by modifying the observational strategy. We take observations uniformly over various intervals but with fixed total observing time, T, and then take three additional observations, each with observing time $T/20$, at 3 months, 9 months, and 12 months after the peak of the event. Thus, the total observing time is $1.15T$. We then find that the precision of both $\tilde{r}_\mathrm{E}$ and θ_E is roughly constant when the continuous observations last anywhere from 30 to 120 days. Any choice in this range would lead to essentially the same result. We adopt 50 days, since it is away from the edges of the interval but still on the shorter side (thus keeping the observations away from the time when the Sun comes close to bulge fields).

4.3. *S/N: Assumptions and Scalings*

We assume that all bulge sources have magnitude $I = 15$, which is typical of the brighter microlensing events seen toward the bulge. For example, of the 143 bulge events alerted by the MACHO Collaboration[1] during 1997, 1998, and the first part of 1999, eight had $I \leq 15.5$ (assuming $V-R = R-I$). Three of the 13 bulge events alerted by the OGLE Collaboration[2] in the first part of 1999 had $I \leq 15.5$. In our calculations, we normalize the astrometric precision by assuming that 4 μas accuracy (in one dimension) can be achieved in 1 minute of observation on an $I = 11$ star. That is, our fiducial $I = 15$ stars require 40 minutes to reach 4 μas. We allow a total of 5 hr of observation for each event. It is then straightforward to scale our results to other assumed conditions. For example, for an $I = 20$ source, the errors reported in the next section must be multiplied by 10. Alternatively, the same errors could be achieved by allowing 500 hr of observations. If our astrometric error estimate proves too pessimistic, so that it is possible to achieve 4 μas precision in a minute on an $I = 12$ star, then the errors should be divided by 1.6.

Toward the LMC, we assume that the source is $V = 20$, which is near the bright end of the events detected in this direction. Because the LMC sources are fainter and the LMC events are rarer than those seen toward the bulge, we assume a total *SIM* integration time of 20 hr. Note that since about 50 times more photons are received from an $I = 15$ star than from a $V = 20$ star, and since we have assumed a fourfold increase in integration time toward the LMC, photon statistics alone will produce $(50/4)^{1/2} \sim 3.5$ times larger errors toward the LMC compared to the bulge. There will be additional differences due to the different geometries.

5. RESULTS

5.1. *Bulge*

We consider two geometries. The first is a bulge line of sight at 6° from the ecliptic. The source and lens are both in the bulge, with $D_\mathrm{os} = 8$ kpc and $D_\mathrm{ol} = 6$ kpc. Hence $D = 24$ kpc. (See eq. [2].) The speed of the lens relative to the observer-source line of sight is $v = 200\ \mathrm{km\ s^{-1}}$. We vary M, ϕ, and β. Formally β is defined as the impact parameter of the event as seen from an observer at the Earth-*SIM* midpoint, but in practice it is very similar to the β observed from the Earth. All events are assumed to peak (as seen from the midpoint) on May 7, i.e., 45 days before the (northern) summer solstice. This is the most favorable time to break the discrete degeneracy in $\Delta\beta$ photometrically (see Gaudi & Gould 1997), but the intrinsic errors in $\Delta\beta$ and Δt_0 are larger by $\sim 2^{1/2}$ than they would be at the summer solstice. We will therefore later investigate whether the discrete degeneracy in $\Delta\beta$ can be broken astrometrically so that observations could take place at the solstice. In order to better understand this and several other issues, we conduct two sets of simulations, one using *SIM* measure-

[1] See http://darkstar.astro.washington.edu/.

[2] See http://www.astrouw.edu.pl/~ftp/ogle/ogle2/ews/ews.html.

ments alone (both astrometric and photometric) and the other combining astrometry with photometry from both *SIM* and the ground.

Table 1 shows the results. The data are grouped in sections for each pair of input parameters M and β. The first column gives the input parameter ϕ. Columns (2) and (3) show the errors in ϕ from *SIM* measurements only and with the addition of ground-based measurements. Columns (4)–(9) show the fractional errors in various quantities both without and with ground-based photometry. Columns (4) and (5) show the errors in $\tilde{r}_E$, columns (6) and (7) show the errors in θ_E, and columns (8) and (9) show the errors in $M = \tilde{r}_E \theta_E c^2/4G$. Finally, columns (10) and (11) show the fractional errors in $D = \tilde{r}_E/\theta_E$, and in π_s based on combined data from *SIM* and the ground.

These results are in rough qualitative agreement with the predictions of equations (10) and (18). In particular, they confirm that the fractional error in $\tilde{r}_E$ is orders of magnitude larger than the fractional error in $\tilde{\theta}_E$ if one is restricted to *SIM* data, while the two errors are comparable if one combines astrometric with photometric data.

However, there are a number of additional important conclusions that can be drawn from Table 1. First, restricting consideration to impact parameters $\beta < 0.5$, the fractional errors in M, D, and π_s are all typically about 5%, although these errors do vary somewhat in particular cases. This means 5 hr of observation produce very precise individual solutions for the mass, distance, and velocity of the lens, and also for the distance and velocity of the source, implying that a few hundred hours of *SIM* time could yield a very detailed inventory of the material between the Sun and the Galactic center.

Second, while the errors in $\tilde{r}_E$ do deteriorate toward $\phi = 90°$, the trend is not as drastic as predicted by equation (18). For $\beta < 0.5$, the errors are 50%–100% worse at $\phi = 90°$ compared to $\phi = 0$, although equation (18) predicts that they should be infinite. The fundamental reason for this is that the continuous degeneracy in $\Delta\beta$ is not very severe, so that if one assumes that the discrete degeneracy is broken, then there is actually quite a lot of information about this component of $\tilde{r}_E$ in the photometric measurements. In fact, the values in Table 1 implicitly *do assume* that the discrete degeneracy is broken. This is because they are based on equation (25), which gives a *purely local* error analysis.

Recall that there are two discrete degeneracies. We focus initially on the one that affects the *magnitude* of $\Delta\beta$ and defer consideration of the one that affects only the sign of $\Delta\beta$. To determine whether this discrete degeneracy is broken photometrically, we examine the work of Gaudi & Gould (1997), in particular their Figure 6. Under the observational conditions they considered, the discrete degeneracy is broken 90% of the time for $M = 0.3\ M_\odot$, $\psi = 225°$, and satellite separations $d_{\oplus\text{-}s} = 0.4$AU. This is twice the separation that we have assumed. However, Gaudi & Gould (1997) have assumed photometry errors of 1% for the Earth and 2% for the satellite, for a total of about 70 observations. If our 5 hr of observing time were divided among 70 observations, the photometric precision would be 1.2%. That is, our assumed effective errors are smaller by a factor $[(1.2^2 + 1.2^2)/(1^2 + 2^2)]^{1/2} = 0.75$. For small Earth-satellite separations, there is a direct trade-off between measurement error and satellite separation, so our 0.2 AU separation corresponds to 0.25 AU in their simulations. Inspection of the Gaudi & Gould (1997) Figure 6 shows that the degeneracies would be broken 70% of the time. To determine how this effectiveness scales with lens mass, we turn to Figure 4 of Gaudi & Gould (1997). This shows that degeneracy breaking becomes more difficult at lower masses. The figure is drawn for the case $d_{\oplus\text{-}s} = 1$ AU, whereas the argument just given implies that with 5 hr of observation, our 0.2 AU separation is equivalent to 0.25 AU in the Gaudi & Gould (1997) simulations. Hence, comparing the Gaudi & Gould (1997) Figures 4 and 6, we estimate that 35% of the degeneracies would be broken for $M = 0.1\ M_\odot$. If the exposure times were multiplied by a factor of 4–20 hr, this fraction would rise to about 70%.

We now turn to the question of how well the degeneracies can be broken astrometrically. For the geometry considered here, $\tilde{r}_E = 7.5(M/0.3\ M_\odot)$AU. Hence $\Delta\beta \sim d_{\oplus\text{-}s}/\tilde{r}_E \sim 0.025$, which is quite small compared to typical values of β. This means that for most events the discrete degeneracy will be between solutions with $\Delta\beta \sim 0.025$ and $\Delta\beta \simeq 2\beta \sim 0.5$. Since $\tan\phi = \Delta\beta/\Delta t_0$, the high $\Delta\beta$ solution will almost always correspond to angles $90° \pm 2°$ or $270° \pm 2°$, while the low $\Delta\beta$ solution (assuming that it is the real one) will be at some random angle. Thus, to distinguish the two solutions, one must have independent information about ϕ with errors that are a factor of ~ 3 smaller than $|90° - \phi|$ or $|270° - \phi|$. Is ϕ this well constrained by the observations? Looking at column (2) of Table 1, ϕ seems to be very well constrained. However, this precision measurement is based primarily on the photometric measurements of $\Delta\beta$ and Δt_0 and so implicitly assumes that the discrete degeneracy has been broken. Hence we should use only *SIM* data (col. [2]).

We see from column (2) of Table 1 that the errors in ϕ are generally small for $\phi \leq 60°$ but deteriorate toward $\phi = 90°$. This means that the discrete degeneracy is broken astrometrically at the 3 σ level for $\phi \leq 60°$ but cannot be broken if the angle gets close enough to 90°. Table 1 does not have sufficient resolution to determine the transition, but we find by more detailed calculations that for $\beta = (0.2, 0.4)$ this occurs at (70°, 65°), for $M = 0.1\ M_\odot$, at (75°, 70°), for $M = 0.3\ M_\odot$, and at (80°, 75°), for $M = 0.5\ M_\odot$. Hence, the degeneracy is usually broken astrometrically, but less frequently at low masses. Since the degeneracy is more difficult to break at low masses both photometrically and astrometrically, it would be prudent to commit more observation time (say 20 hr rather than 4 hr) to the shortest events (which are likely to be low mass).

The fact that the degeneracy can be broken astrometrically for most events seems to argue against restricting observations to periods that are 45 days from the summer solstice. Recall that we adopted this restriction in order to permit better photometric degeneracy breaking, which now no longer seems so necessary. However, we find that for events peaking at the solstice, the errors in ϕ (when the Earth-based observations are ignored; col. [2]) are substantially higher than the values in column (3), implying that it is often not possible to break the discrete degeneracy astrometrically or photometrically at the solstice.

As we have noted, the above discussion actually applies to only one of two discrete degeneracies, the one involving two different *magnitudes* of $\Delta\beta$. This is the more important degeneracy because it affects the estimate of the size of $\tilde{r}_E$ and so of the mass, distance, and speed of the lens. However, there is also another degeneracy involving the *sign* of $\Delta\beta$ but not its magnitude. Boutreux & Gould (1996) and Gaudi

TABLE 1

UNCERTAINTIES OF PARAMETERS: BULGE ($I = 15$, $\psi = 225°$, 5 hr)

	σ_ϕ		$\sigma_{\tilde{r}_E}/\tilde{r}_E$		$\sigma_{\theta_E}/\theta_E$		σ_M/M			
ϕ (deg) (1)	S (deg) (2)	S+ ⊕ (deg) (3)	S (4)	S+ ⊕ (5)	S (6)	S+ ⊕ (7)	S (8)	S+ ⊕ (9)	σ_D/D S+ ⊕ (10)	σ_{π_s}/π_s S+ ⊕ (11)
				$M = 0.1\ M_\odot$, $\beta = 0.2$						
0	2.3	1.1	0.981	0.015	0.041	0.041	0.980	0.044	0.043	0.035
30	2.5	1.0	1.265	0.017	0.041	0.041	1.265	0.045	0.044	0.035
60	4.3	0.9	2.713	0.020	0.040	0.040	2.713	0.046	0.045	0.034
90	9.2	0.7	6.005	0.022	0.040	0.039	6.012	0.045	0.045	0.033
				$M = 0.1\ M_\odot$, $\beta = 0.4$						
0	2.7	2.2	1.962	0.036	0.046	0.045	1.955	0.059	0.057	0.035
30	3.5	2.1	2.564	0.041	0.047	0.046	2.559	0.063	0.061	0.036
60	7.9	1.9	5.394	0.056	0.046	0.046	5.394	0.075	0.071	0.036
90	17.0	1.6	11.382	0.070	0.052	0.044	11.408	0.084	0.082	0.035
				$M = 0.1\ M_\odot$, $\beta = 0.6$						
0	4.0	3.0	4.271	0.075	0.060	0.053	4.248	0.095	0.088	0.036
30	6.9	2.9	5.500	0.080	0.060	0.055	5.482	0.100	0.094	0.036
60	16.5	2.7	11.109	0.111	0.057	0.055	11.108	0.128	0.121	0.037
90	27.0	2.5	18.428	0.173	0.084	0.054	18.491	0.185	0.177	0.036
				$M = 0.3\ M_\odot$, $\beta = 0.2$						
0	1.1	0.9	0.446	0.019	0.021	0.021	0.446	0.029	0.028	0.037
30	1.2	0.9	0.570	0.021	0.021	0.021	0.570	0.030	0.030	0.037
60	2.0	0.9	1.195	0.026	0.020	0.020	1.196	0.034	0.033	0.036
90	4.3	0.8	2.768	0.030	0.019	0.019	2.772	0.035	0.035	0.035
				$M = 0.3\ M_\odot$, $\beta = 0.4$						
0	1.3	1.2	0.900	0.047	0.024	0.024	0.899	0.053	0.052	0.038
30	1.7	1.2	1.132	0.054	0.024	0.024	1.132	0.059	0.058	0.038
60	3.5	1.2	2.267	0.075	0.023	0.023	2.270	0.079	0.077	0.038
90	8.5	1.2	5.639	0.097	0.026	0.022	5.653	0.100	0.100	0.037
				$M = 0.3\ M_\odot$, $\beta = 0.6$						
0	2.0	1.6	1.906	0.098	0.030	0.029	1.899	0.103	0.101	0.038
30	3.2	1.6	2.321	0.109	0.030	0.029	2.317	0.114	0.112	0.038
60	6.8	1.6	4.280	0.158	0.029	0.029	4.284	0.162	0.159	0.038
90	14.8	1.6	9.849	0.252	0.047	0.027	9.888	0.254	0.252	0.038
				$M = 0.5\ M_\odot$, $\beta = 0.2$						
0	0.9	0.8	0.313	0.022	0.017	0.017	0.313	0.027	0.027	0.038
30	0.9	0.8	0.395	0.024	0.016	0.016	0.396	0.029	0.029	0.038
60	1.5	0.8	0.820	0.031	0.015	0.015	0.821	0.034	0.034	0.037
90	3.0	0.8	1.999	0.035	0.015	0.015	2.002	0.038	0.038	0.036
				$M = 0.5\ M_\odot$, $\beta = 0.4$						
0	1.0	1.0	0.623	0.054	0.019	0.019	0.623	0.057	0.057	0.039
30	1.3	1.0	0.783	0.062	0.019	0.019	0.784	0.065	0.065	0.039
60	2.4	1.0	1.536	0.089	0.018	0.018	1.539	0.091	0.091	0.038
90	6.5	1.0	4.352	0.115	0.020	0.016	4.362	0.115	0.116	0.038
				$M = 0.5\ M_\odot$, $\beta = 0.6$						
0	1.5	1.2	1.386	0.113	0.023	0.023	1.384	0.116	0.115	0.040
30	2.3	1.2	1.633	0.129	0.023	0.023	1.634	0.132	0.131	0.040
60	4.6	1.3	2.903	0.195	0.023	0.022	2.909	0.197	0.196	0.039
90	12.3	1.3	8.190	0.293	0.039	0.020	8.224	0.294	0.293	0.039
				$M = 1.0\ M_\odot$, $\beta = 0.2$						
0	0.7	0.6	0.211	0.026	0.014	0.013	0.212	0.029	0.029	0.040
30	0.7	0.7	0.266	0.029	0.013	0.013	0.267	0.032	0.032	0.039
60	1.0	0.7	0.540	0.038	0.012	0.012	0.541	0.039	0.040	0.039
90	2.1	0.7	1.458	0.044	0.012	0.012	1.460	0.044	0.046	0.038

TABLE 1—*Continued*

ϕ (deg) (1)	σ_ϕ S (deg) (2)	σ_ϕ S+ $\oplus$ (deg) (3)	$\sigma_{\tilde{r}_E}/\tilde{r}_E$ S (4)	$\sigma_{\tilde{r}_E}/\tilde{r}_E$ S+ $\oplus$ (5)	$\sigma_{\theta_E}/\theta_E$ S (6)	$\sigma_{\theta_E}/\theta_E$ S+ $\oplus$ (7)	σ_M/M S (8)	σ_M/M S+ $\oplus$ (9)	σ_D/D S+ $\oplus$ (10)	σ_{π_s}/π_s S+ $\oplus$ (11)
					$M = 1.0\ M_\odot$, $\beta = 0.4$					
0.......	0.7	0.7	0.410	0.065	0.016	0.015	0.412	0.066	0.067	0.041
30......	0.9	0.8	0.502	0.077	0.016	0.015	0.504	0.078	0.079	0.040
60......	1.5	0.8	0.963	0.115	0.015	0.014	0.967	0.115	0.116	0.040
90......	4.6	0.9	3.268	0.137	0.016	0.013	3.276	0.136	0.139	0.040
					$M = 1.0\ M_\odot$, $\beta = 0.6$					
0.......	1.0	1.0	0.975	0.137	0.019	0.018	0.979	0.138	0.138	0.042
30......	1.5	1.0	1.122	0.164	0.019	0.018	1.126	0.166	0.165	0.042
60......	2.9	1.1	1.908	0.262	0.019	0.016	1.916	0.262	0.262	0.041
90......	10.2	1.1	6.937	0.333	0.034	0.015	6.967	0.333	0.334	0.041

NOTE.—M is the mass of the lens, β is the impact parameter in Einstein radius units, ϕ is the angle of lens motion relative to the satellite-Earth vector. "S" and "S+ $\oplus$" designate two cases: first, *SIM* astrometry and photometry measurements; second, *SIM* measurements plus ground-based photometry. $\tilde{r}_E$ is the projected Einstein radius (eq. [3]), θ_E is the angular Einstein radius (eq. [2]), $D = \tilde{r}_E/\theta_E$ is given in eq. (2), and π_s is the parallax of the source.

& Gould (1997) refer to the first of these as the "speed degeneracy." We call the second the "direction degeneracy." The direction degeneracy becomes difficult to break when $|\Delta\beta| \ll |\Delta t_0|/t_E$, i.e., when $\phi \sim 0°$ or $\phi \sim 180°$, so that the two degenerate solutions are close in ϕ. From column (2) in Table 1, we find that the error in ϕ from the astrometric data alone is generally quite small for $\phi = 0$, and hence is adequate to break the direction degeneracy unless ϕ lies within a few degrees of either 0° or 180°. However, in this case the effect of the degeneracy is very small.

Finally, we note that we have examined the errors in F_b/F_s (although we do not display them). We find that they are typically a few percent, implying that a luminous lens could be detected if it were more than a few percent of the flux of the source.

5.2. *LMC*

The second line of sight is toward the LMC at the south ecliptic pole. The source lies at $D_{os} = 50$ kpc, while the lens is assumed to be in the halo at $D_{ol} = 15$ kpc. Hence $D = 21$ kpc, which is very similar to the bulge value. This means that at fixed mass, the bulge events considered in § 5.1 will have about the same $\tilde{r}_E$ as the LMC events considered here. The speed of the lens relative to the observer-source line of sight is $v = 250$ km s^{-1}, slightly larger than for the bulge. Recall that we are assuming that the source is $V = 20$ and that the total observing time is 20 hr.

Table 2 shows the results. Apart from the factor of $\sim$3.5 larger errors that results simply from photon statistics, they are qualitatively similar to those for the bulge. The largest difference is that the fractional error in π_s is larger, which simply reflects the fact that the LMC is more distant.

Of the eight microlensing events detected by Alcock et al. (1997b) during their first two years of observations, none of the sources were brighter than $V = 20$ (after removing blended light), which is the nominal limit for *SIM*. Future microlensing surveys could improve the rate of detection by an order of magnitude (Gould 1999b; Stubbs 1998). However, only a factor of 3 of this improvement would be due to the coverage of a larger area. The rest would come from going deeper, which would not yield any more bright sources. Hence, the total rate of events that are accessible to *SIM* will not be high. Most of the usable events that are detected are likely to be close to the magnitude limit. We find that with our assumed source magnitudes and exposure times, the mass and distance estimates will be accurate to about 10%–20% (although they rise to $\sim$40% for the case of $M = M_\odot$, $\beta = 0.4$). This would be an acceptable level of precision to resolve the question of the nature of the lenses assuming that more than a handful of events can be measured. The errors in the measurement of ϕ without making use of the Earth-based observations (col. [2]) are typically 8°. Hence in many cases it will not be possible to break the $\Delta\beta$ degeneracy astrometrically.

To determine whether the photometry is sufficiently precise to break the degeneracy, we compare our simulation with that of Boutreux & Gould (1996), who specifically considered an Earth-satellite separation of 0.26 AU—close to our value of $d_{\oplus\text{-}s} = 0.2$ AU. In their Monte Carlo simulation, the "speed degeneracy" (between different scalar values of $\Delta\beta$) was broken 40%–60% of the time in the mass range 0.1–1 $M_\odot$. We find that their assumed photometric errors are about twice the ones assumed here. Therefore, it seems likely that the *SIM* photometric observations would be adequate to break this degeneracy in the majority of cases.

Inspection of column (2) of Table 2 shows that the direction degeneracy will usually be broken astrometrically. The simulations of Boutreux & Gould (1996) show that it is about as difficult to break the direction degeneracy as the speed degeneracy. Thus, it should usually also be possible to break this degeneracy photometrically. In any event, as in the case of the bulge, the direction degeneracy is much less important than the speed degeneracy.

Of course, not all halo lenses can be expected to be at $D_{ol} = 15$ kpc. We also considered $D_{ol} = 10$ kpc and $D_{ol} = 25$ kpc. For the first case, we find that the fractional errors are smaller than those in Table 2 by a factor of $\sim$0.8 for $M \leq 0.5\ M_\odot$ and $\sim$0.5 for $M \sim M_\odot$ (except for σ_{π_s}, which

TABLE 2

UNCERTAINTIES OF PARAMETERS: LMC ($V = 20$, arbitrary ψ, 20 hr)

ϕ (deg) (1)	σ_ϕ S (deg) (2)	σ_ϕ S+ $\oplus$ (deg) (3)	$\sigma_{\tilde{r}_E}/\tilde{r}_E$ S (4)	$\sigma_{\tilde{r}_E}/\tilde{r}_E$ S+ $\oplus$ (5)	$\sigma_{\theta_E}/\theta_E$ S (6)	$\sigma_{\theta_E}/\theta_E$ S+ $\oplus$ (7)	σ_M/M S (8)	σ_M/M S+ $\oplus$ (9)	σ_D/D S+ $\oplus$ (10)	σ_{π_s}/π_s S+ $\oplus$ (11)
$M = 0.1\ M_\odot$, $\beta = 0.2$										
0	6.7	2.5	5.353	0.032	0.145	0.145	5.356	0.145	0.145	0.827
30	7.1	2.1	1.124	0.035	0.138	0.138	1.128	0.141	0.141	0.788
60	7.8	1.8	0.580	0.046	0.124	0.120	0.587	0.127	0.127	0.689
90	8.5	1.4	0.438	0.049	0.110	0.110	0.449	0.120	0.120	0.626
$M = 0.1\ M_\odot$, $\beta = 0.4$										
0	7.4	5.7	8.326	0.078	0.166	0.159	8.362	0.180	0.177	0.852
30	8.8	5.7	2.302	0.092	0.159	0.156	2.316	0.187	0.177	0.834
60	9.5	5.3	1.234	0.134	0.141	0.138	1.241	0.202	0.184	0.757
90	10.3	3.9	0.955	0.166	0.124	0.124	0.955	0.205	0.212	0.672
$M = 0.1\ M_\odot$, $\beta = 0.6$										
0	8.8	8.1	11.455	0.170	0.219	0.191	11.561	0.262	0.247	0.880
30	13.4	8.5	5.675	0.187	0.205	0.191	5.749	0.279	0.255	0.877
60	14.1	9.2	3.362	0.279	0.177	0.170	3.405	0.350	0.301	0.849
90	14.8	8.1	2.634	0.456	0.148	0.148	2.641	0.481	0.477	0.767
$M = 0.3\ M_\odot$, $\beta = 0.2$										
0	4.2	2.8	2.507	0.039	0.110	0.110	2.496	0.117	0.117	0.891
30	4.6	2.8	0.707	0.046	0.106	0.099	0.707	0.110	0.110	0.831
60	6.0	2.5	0.357	0.057	0.088	0.081	0.361	0.103	0.099	0.697
90	6.7	2.1	0.272	0.064	0.074	0.074	0.286	0.095	0.099	0.612
$M = 0.3\ M_\odot$, $\beta = 0.4$										
0	4.9	4.6	3.843	0.117	0.124	0.124	3.850	0.163	0.173	0.933
30	5.7	4.9	1.245	0.131	0.117	0.113	1.245	0.177	0.170	0.912
60	7.1	5.7	0.654	0.177	0.099	0.095	0.658	0.205	0.194	0.845
90	8.1	4.9	0.520	0.219	0.085	0.085	0.527	0.226	0.240	0.750
$M = 0.3\ M_\odot$, $\beta = 0.6$										
0	6.0	5.7	5.218	0.279	0.152	0.145	5.271	0.311	0.318	0.983
30	7.8	6.4	2.581	0.311	0.138	0.134	2.606	0.343	0.332	0.969
60	9.9	8.1	1.556	0.442	0.113	0.113	1.566	0.467	0.445	0.969
90	11.0	8.8	1.255	0.665	0.103	0.099	1.241	0.651	0.689	0.940
$M = 0.5\ M_\odot$, $\beta = 0.2$										
0	4.2	3.2	2.524	0.049	0.113	0.106	2.496	0.117	0.117	0.912
30	4.6	3.2	0.841	0.053	0.103	0.095	0.845	0.113	0.110	0.856
60	5.7	2.8	0.467	0.071	0.085	0.078	0.474	0.106	0.103	0.707
90	6.4	2.5	0.378	0.078	0.071	0.067	0.389	0.103	0.106	0.612
$M = 0.5\ M_\odot$, $\beta = 0.4$										
0	4.6	4.2	3.345	0.156	0.124	0.120	3.330	0.191	0.205	0.972
30	5.7	4.9	1.107	0.173	0.113	0.110	1.114	0.212	0.202	0.955
60	7.1	6.0	0.633	0.233	0.092	0.088	0.640	0.255	0.244	0.894
90	7.8	5.7	0.527	0.265	0.078	0.078	0.534	0.272	0.279	0.831
$M = 0.5\ M_\odot$, $\beta = 0.6$										
0	5.3	4.9	3.737	0.403	0.145	0.141	3.769	0.421	0.435	1.025
30	7.4	6.4	1.824	0.445	0.127	0.127	1.849	0.470	0.456	1.018
60	9.5	8.5	1.177	0.587	0.103	0.099	1.191	0.601	0.590	1.043
90	10.6	9.2	1.022	0.682	0.095	0.092	1.011	0.668	0.707	1.022
$M = 1.0\ M_\odot$, $\beta = 0.2$										
0	3.9	3.5	1.881	0.166	0.110	0.106	1.856	0.194	0.202	0.898
30	4.6	3.5	0.537	0.198	0.099	0.095	0.534	0.219	0.223	0.852
60	5.7	4.6	0.297	0.152	0.081	0.078	0.301	0.159	0.184	0.810
90	6.4	5.3	0.233	0.110	0.067	0.067	0.240	0.127	0.131	0.806

TABLE 2—*Continued*

ϕ (deg) (1)	σ_ϕ S (deg) (2)	σ_ϕ S+ ⊕ (deg) (3)	$\sigma_{\tilde{r}_E}/\tilde{r}_E$ S (4)	$\sigma_{\tilde{r}_E}/\tilde{r}_E$ S+ ⊕ (5)	$\sigma_{\theta_E}/\theta_E$ S (6)	$\sigma_{\theta_E}/\theta_E$ S+ ⊕ (7)	σ_M/M S (8)	σ_M/M S+ ⊕ (9)	σ_D/D S+ ⊕ (10)	σ_{π_s}/π_s S+ ⊕ (11)
					$M = 1.0\ M_\odot$, $\beta = 0.4$					
0.......	4.2	4.2	2.630	0.424	0.120	0.120	2.648	0.445	0.438	0.997
30......	6.0	5.7	0.725	0.385	0.110	0.110	0.739	0.403	0.396	0.997
60......	7.4	6.7	0.424	0.279	0.085	0.081	0.431	0.290	0.297	0.965
90......	7.8	7.1	0.350	0.237	0.074	0.074	0.357	0.244	0.255	0.937
					$M = 1.0\ M_\odot$, $\beta = 0.6$					
0.......	4.6	4.6	2.956	0.940	0.145	0.134	3.016	0.969	0.933	1.057
30......	7.8	7.1	1.280	0.788	0.124	0.120	1.326	0.827	0.771	1.138
60......	9.9	9.5	0.810	0.612	0.088	0.088	0.824	0.622	0.612	1.142
90......	9.5	9.2	0.693	0.541	0.088	0.088	0.689	0.527	0.562	1.057

is unaffected). For the second case, we find that these errors are larger than those in Table 2 by a factor of ~ 1.6 for $M \leq 0.5\ M_\odot$ and ~ 3 for $M \sim M_\odot$. Thus, the results reported in Table 2 apply qualitatively to a broad range of halo distances for $M \leq 0.5\ M_\odot$ but not for $M \sim M_\odot$.

If the lenses detected toward the LMC are in the LMC itself (rather than in the halo), then we find that neither $\tilde{r}_E$ nor θ_E can be detected, let alone measured, in our fiducial 20 hr of observation. However, even nondetections of these two quantities would be highly significant, as it would demonstrate that the lens was in the LMC.

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