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A NEW KINEMATIC DISTANCE ESTIMATOR TO THE LARGE MAGELLANIC CLOUD

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ABSTRACT

The distance to the Large Magellanic Cloud (LMC) can be directly determined by measuring three of its properties: its radial velocity field, its mean proper motion, and the position angle ϕ_{ph} of its photometric line of nodes. Statistical errors of $\sim 2\%$ are feasible based on proper motions obtained with any of several proposed astrometry satellites, the earliest possibility being the *Full-Sky Astrometric Mapping Explorer* (*FAME*). The largest source of systematic error is likely to be in the determination of ϕ_{ph}. I suggest two independent methods to measure ϕ_{ph}, one based on counts of clump giants and the other on photometry of clump giants. I briefly discuss a variety of methods to test for other sources of systematic errors.

Subject headings: astrometry — galaxies: distances and redshifts — Magellanic Clouds

1. INTRODUCTION

The distance to the Large Magellanic Cloud (LMC) plays a crucial role in the extragalactic distance scale. The relation between log period and apparent magnitude of LMC Cepheids is quite well determined. If an LMC distance d_{LMC} and mean LMC Cepheid reddening are assumed, then the Cepheid period-luminosity relation is effectively calibrated. The distance to external galaxies harboring Cepheids can then be determined by comparing their observed fluxes to those of LMC Cepheids at the same period, and by taking account of the differences in reddening that are determined from the differences in color between the target Cepheids and those in the LMC. A variety of secondary distance indicators have been calibrated in this fashion. While a decade ago, the Hubble constant H_0 derived from these measurements ranged over a factor of 2 depending strongly on both the author and the method, a major observing campaign with the *Hubble Space Telescope* (*HST*) has dramatically narrowed this conflict. For example, (Saha et al. 1999) recently find $H_0 d_{LMC}/50$ kpc $= 60 \pm 2$ (internal) km s^{-1} Mpc^{-1}, while Madore et al. (1999) find $H_0 d_{LMC}/50$ kpc $= 72 \pm 3$ (random) $\pm$ 5 (systematic) km s^{-1} Mpc^{-1}.

By contrast, the disagreements over d_{LMC} have not narrowed at all over the past decade. The primary methods for measuring d_{LMC} use "standard candles," objects whose luminosity is presumed to be fixed or to depend only on distance-independent observables such as period, metallicity, etc. Their absolute magnitudes must be calibrated locally. Two major standard candles that have been used to measure d_{LMC} are Cepheids and RR Lyrae stars. Three recent determinations, all from *Hipparcos*-based calibrations of these standard candles, illustrate the range of d_{LMC} estimates: Feast & Catchpole (1997) find $d_{LMC} = 55.0 \pm 2.5$ kpc based on trigonometric parallaxes of Cepheids; Gould & Popowski (1998) find $d_{LMC} = 45.1 \pm 2.7$ kpc based on statistical parallax of RR Lyrae stars [and assuming V_0(RR, LMC) = 18.98; Walker 1992]; and Gratton et al. (1997) finds $d_{LMC} = 52.1 \pm 1.7$ kpc from an RR Lyrae calibration based on fitting globular cluster main sequences to *Hipparcos* subdwarfs, while Reid (1997) finds a slightly longer distance based on the same technique. At present it is not known whether these discrepancies are due to undetected systematic errors in the various techniques or to nonstandardness in one or more of the "standard candles," or both. It is unlikely that the differences are merely statistical fluctuations. For example, Popowski & Gould (1999) review a variety of methods for calibrating RR Lyrae stars whose results disagree by substantially more than their statistical errors.

Of course, one would prefer to eliminate the distance ladder altogether and simply obtain a direct measurement of d_{LMC} There are two possible paths to a direct distance measurement: trigonometric parallax and kinematic methods. The parallax of the LMC is $\pi \sim 20$ μas. The *Space Interferometry Mission* (*SIM*) should be able to make individual astrometric measurements accurate to ~ 8 μas, and could perhaps achieve $\sigma_\pi \sim 2$ μas given a sufficient number of observations. This limit is set by the precision of the *SIM* "grid-star" solution. Hence it cannot be significantly improved upon by making measurements of several LMC stars, since these lie in the same field. While such a $\sim 10\%$ measurement would certainly be of interest, it would not by itself clearly distinguish among the various competing distance estimates.

Panagia et al. (1991) made the first kinematic measurement of d_{LMC} by comparing the light-travel time across the ring around SN 1987A with its angular diameter as measured by *HST*. They found $\mu_{SN} = 18.55 \pm 0.13$, where μ_{SN} is the distance modulus of SN 1987A. Gould (1995b) reanalyzed these data and obtained $\mu_{SN} \leq 18.350 \pm 0.035$. (Sonneborn et al. 1997) rereduced the original light-curve data and found $\mu_{SN} = 18.43 \pm 0.10$. Gould & Uza (1998) then reanalyzed these rereduced data and obtained $\mu_{SN} \leq 18.372 \pm 0.035$ if the ring were assumed circular, but $\mu_{SN} \leq 18.44 \pm 0.05$ if it were assumed elliptical (as some evidence suggests). Finally, Panagia (1998), using the same data, but arguing that the effective radius of the ring had grown between the time of the light-echo measurements and those of the angular size of the ring, found $\mu_{SN} = 18.55 \pm 0.05$. In brief, there remains controversy over the interpretation of the data at the $\sim 10\%$ level in distance. Since the event itself was unique and the measurements will never be repeated, it seems unlikely that this conflict will be resolved to everyone's satisfaction.

Here I propose to use the radial velocity gradient method to measure the distance to the LMC. This method has been used in the past to measure the distance to the Hyades (Detweiler et al. 1984; Gunn et al. 1988) and the Pleiades (Narayanan & Gould 1999). When applied to the LMC, the method has some unique characteristics relative to previous applications. This is in part because the LMC is a cold system supported primarily by rotation while the Hyades and Pleiades are supported by pressure, and in part because the LMC is 2 orders of magnitude farther away.

Consider a cold disk rotating at a projected angular rate $\Omega(R)$, and moving with a systemic proper motion $\boldsymbol{\mu}$ (and so transverse velocity $\boldsymbol{V}_\perp = \boldsymbol{\mu} d$). If $\mu \ll \Omega$, then the locus of extrema in the radial velocity field will coincide with the photometric major axis. That is, the kinematic and photometric lines of nodes will be aligned. However, the transverse velocity $\boldsymbol{V}_\perp$ of any system induces a gradient in the radial velocities because the radial vector that is dotted into the velocity to form the radial velocity changes direction across the system. Thus the observed gradient ∇v_r will be displaced from that due to internal rotation alone by $\boldsymbol{V}_\perp$. If the direction of the photometric line of nodes is known, and if $\boldsymbol{\mu}$ is measured (so that the direction of $\boldsymbol{V}_\perp$ is also known), then it is straightforward to solve for the magnitude of $\boldsymbol{V}_\perp$. The distance is then simply $d = V_\perp/\mu$. The LMC is sufficiently close, and is moving sufficiently rapidly, that the kinematic line of nodes is displaced from the photometric line of nodes by about 25°. The interpretation of this displacement could be clouded by uncertainty about how well the LMC conforms to the ideal of a flat axisymmetric system that I use to model the data. After I present the method and derive the statistical uncertainties, I briefly discuss how the measurement could be corrupted by systematic deviations from this ideal, and I indicate some methods to check for such systematic effects.

To illustrate the method, I will assume the use of astrometry data such as would be obtained by the *Full-Sky Astrometric Explorer* (*FAME*), a proposed Midex mission. As I will discuss, the method could also be applied to data from *SIM* or the *Global Astrometric Interferometer for Astrophysics* (*GAIA*).

2. THE METHOD

Consider a stellar system whose physical size is small compared to its distance d. Let the space velocity of the system be $\boldsymbol{V}$, and let the space motion of an individual star in the system be $\boldsymbol{v}_i$. I then write

$$\boldsymbol{v}_i = \boldsymbol{V} + \boldsymbol{u}_i + \delta \boldsymbol{v}_i \,, \tag{1}$$

where $\boldsymbol{u}_i$ is the mean internal systemic motion of the stars in the system (due, e.g., to rotation) at the projected position of star i, and $\delta \boldsymbol{v}_i$ is the peculiar motion of star i relative to this systemic motion. The radial-velocities are therefore given by

$$v_{r,i} = \boldsymbol{n}_i \cdot \boldsymbol{V} + u_{r,i} + \delta v_{r,i} \,, \tag{2}$$

where $\boldsymbol{n}_i$ is the unit vector in the direction of star i, $u_{r,i} = \boldsymbol{n}_i \cdot \boldsymbol{u}_i$, and $\delta v_{r,i} = \boldsymbol{n}_i \cdot \delta \boldsymbol{v}_i$. I assume that the radial velocity residuals $\delta v_{r,i}$ are randomly distributed with dispersion σ_v. I also assume that the internal systemic motion $\boldsymbol{u}_i$ is known. In fact, determining $\boldsymbol{u}$ is not trivial, but I ignore this problem here and return to it in §§ 3.2, 3.3, and 4. Then the radial velocity gradient with respect to angular position on the sky is given by

$$\nabla v_r = \boldsymbol{V}_\perp + \nabla u_r \,, \tag{3}$$

where $\boldsymbol{V}_\perp = \boldsymbol{V} - \boldsymbol{n}_0(\boldsymbol{n}_0 \cdot \boldsymbol{V})$ is the transverse velocity of the center of the system, and $\boldsymbol{n}_0$ is the direction vector pointing to this center. Since ∇v_r is a vector, the errors are properly described by a covariance matrix, c_{xy}. This is given by

$$c \equiv b^{-1} \,, \quad b_{kl} = \frac{1}{\sigma_v^2}\left[\sum_i \theta_{k,i}\theta_{l,i} - \frac{1}{N_r}\left(\sum_i \theta_{k,i}\right)\left(\sum_i \theta_{l,i}\right)\right] , \tag{4}$$

where $\boldsymbol{\theta} \equiv (\theta_x, \theta_y)$ is the angular position of star i relative to the center, and N_r is the number of radial velocity measurements. However, for simplicity, I will consider stars distributed uniformly over a circular area of radius $\Delta\theta$. In this case, the error in each component of ∇v_r (or equivalently $\boldsymbol{V}_\perp$, since ∇u_r is assumed known) is

$$\sigma(V_\perp) = \sigma_\nabla = c_{kk}^{1/2} = \left(\frac{2}{N_r}\right)^{1/2} \frac{\sigma_v}{\Delta\theta} \,. \tag{5}$$

See also Narayanan & Gould (1999).

Suppose now that the proper motion $\boldsymbol{\mu}$ of the system is measured with error σ_μ The distance and distance error are then

$$d = \frac{V_\perp}{\mu} \,, \quad \left(\frac{\sigma_d}{d}\right)^2 = \left(\frac{\sigma_\nabla}{V_\perp}\right)^2 + \left(\frac{\sigma_\mu}{\mu}\right)^2 . \tag{6}$$

That is, the fractional distance error is limited by the larger of the errors in the transverse velocity and the proper motion.

3. APPLICATION TO THE LMC

3.1. *Naive*

Substituting values appropriate for the LMC in equation (5), I obtain

$$\frac{\sigma_\nabla}{V_\perp} = 1.5\% \left(\frac{N_r}{10^4}\right)^{1/2} \frac{\sigma_v}{24\ \mathrm{km\ s^{-1}}} \left(\frac{V_\perp}{325\ \mathrm{km\ s^{-1}}}\right)^{-1} \left(\frac{\Delta\theta}{4°}\right)^{-1} , \tag{7}$$

where I have chosen a dispersion characteristic of carbon stars (Cowley & Hartwick 1991) and the estimate of the transverse velocity from the proper-motion measurement of Jones, Klemola, & Lin (1994). Hence, good statistical precision is possible provided that a large sample of stars is available. Note that the measurement errors are not important provided that they are well below dispersion. Since $\sigma_v \sim 5$ km s^{-1} errors are not difficult to achieve for LMC carbon stars, it is feasible to obtain a very large sample such as is envisaged in equation (7).

While the proper motion of the LMC is only crudely known today (Jones et al. 1994; Kroupa & Bastian 1997), it could be measured to very high precision with any of a number of proposed astrometry satellites including *FAME*, *SIM*, and *GAIA*. For definiteness, I will focus on the capabilities of *FAME* which has the earliest possibility of launch. I find from the USNO-A2.0 catalog (Monet 1998), that there is a total of 21,900 stars with $13 < V < 15$ within $\Delta\theta = 4°$ of the center of the LMC at $(l, b) = (280°.5, -32°.9)$, where I estimate $V = (B + R)/2$. Of these, about 13,300 are foreground Galactic stars as judged from counts in three similar circles at $(l, b) = (280^\circ.5, +32^\circ.9)$, and $(l, b) = (79^\circ.5, \pm 32^\circ.9)$. This leaves $N_\perp \sim 8600$ stars in the LMC. The dispersions of LMC stars in the transverse directions are unknown, but based on what is known of disk kinematics in the Galaxy, it is plausible to assume that they are $\sim 50\%$ higher than the vertical dispersion, or $\sigma_\perp \sim 35$ km s^{-1}. Hence, if the proper motions of these stars could be measured to better than $\sigma_\perp/d_{\rm LMC} \sim 150$ μas yr^{-1}, and if the internal systemic motions $\boldsymbol{u}$ are again assumed known (see § 4), then the precision of the LMC proper motion would be given by

$$\frac{\sigma_\mu}{\mu} = N_\perp^{-1/2} \frac{\sigma_\perp}{V_\perp} = 0.1\% \left(\frac{N_\perp}{8600}\right)^{-1/2} \frac{\sigma_\perp}{35 \text{ km s}^{-1}} \left(\frac{V_\perp}{325 \text{ km s}^{-1}}\right)^{-1}, \quad (8)$$

where $N_\perp$ is the number of proper-motion measurements. In fact, *FAME* probably cannot achieve quite this precision at $V = 15$, but should come within a factor of 2 (Horner et al. 1998) and so easily achieve $\sigma_\mu/\mu \lesssim 1\%$ or $\sigma_\mu \lesssim 10$ μas yr^{-1}. The present rotational precision of the extragalactic reference frame is $\sigma_\mu \sim 5$ μas yr^{-1}. However, the *FAME* astrometric frame will probably be accurate only to within $\sigma_\mu \sim 25$ μas yr^{-1} (assuming 100 QSOs with $V \lesssim 15$ and hence with mean proper-motion errors of 250 μas yr^{-1}). The *FAME* frame will be fixed substantially better by *SIM*. In brief, the proper-motion measurement error can probably be reduced to about 2% with *FAME* alone and substantially less by combining *FAME* and *SIM*.

3.2. *Degeneracy*

However, the potentially fatal flaw in this method is that $\boldsymbol{u}$ is *not* known (as has been assumed so far) and must be determined from the same kinematic data that are used to derive the distance measurement. As is well known from the classical application of the radial velocity gradient method to the Hyades (Detweiler et al. 1984; Gunn et al. 1988) and the Pleiades (Narayanan & Gould 1999), if the cluster were undergoing solid-body rotation $\boldsymbol{u} = \boldsymbol{\Omega} \times \boldsymbol{r}$, this would produce a radial velocity gradient

$$\nabla u_r = (\boldsymbol{n}_0 \times \boldsymbol{\Omega}) d \,. \quad (9)$$

Here $\boldsymbol{r}$ is the 3-space position of a star relative to the cluster center. This gradient is indistinguishable from the gradient produced by a transverse velocity and so, if unrecognized, would corrupt the distance measurement given by equation (6). In the case of clusters, one normally simply assumes that the cluster is not rotating. However, one can check this assumption by comparing the directions of the radial velocity gradient and the proper motion. If these differ, the cause might be rotation (or systematic errors). If they are the same, then either the cluster is not rotating, or its rotation happens to be perfectly aligned with its proper motion (within statistical errors).

The situation is similar for the LMC but is somewhat more complicated because the LMC *is* rotating. While the rotation is not solid-body, it can be reasonably approximated as such in the inner $2^\circ.5$. To the extent that the rotation is solid-body, one measures a gradient

$$\nabla v_r = \boldsymbol{V}_\perp + \boldsymbol{\Omega}_\times d_{\rm LMC} \,, \quad \boldsymbol{\Omega}_\times \equiv \boldsymbol{n}_0 \times \boldsymbol{\Omega} \,, \quad (10)$$

and from this measurement alone has no idea how to separate the two components. If, for example, one ignored the transverse motion, one would interpret the gradient as due entirely to rotation and would therefore misjudge the amplitude of rotation. One would misjudge its orientation as well to the extent that $\boldsymbol{V}_\perp$ does not happen to lie parallel to $\boldsymbol{\Omega}_\times$.

3.3. *Breaking the Degeneracy*

However, for a disk rotating about its axis of symmetry, $\boldsymbol{\Omega}_\times$ should be aligned with the apparent major axis of the system, i.e., the photometric line of nodes. This provides some information with which to break the degeneracy.

These effects were first investigated when Feitzinger, Isserstedt, & Schmidt-Kaler (1977) reanalyzed earlier kinematic data. They noted that the kinematic line of nodes (locus of extrema in radial velocity) was displaced by $\sim 20^\circ$ from the photometric line of nodes (major axis of the surface brightness profile) at position angle $\phi_{\rm ph} = -10^\circ$. They assumed that this displacement was caused by transverse motion in the direction $\phi_\mu = 110^\circ$ (i.e., the direction of the Magellanic stream) and then solved for the amplitude of this motion $V_\perp \sim 275$ km s^{-1}. Subsequently, several other workers applied a similar procedure to various stellar samples and obtained various results (Rohlfs et al. 1984; Meatheringham, Schwarz, & Murray 1988; Hughes, Wood, & Reid 1991). Note that this approach to breaking the degeneracy requires *two* pieces of information in addition to the kinematic data: first, the position angle of the photometric line of nodes $\phi_{\rm ph}$, and, second, the direction of LMC motion ϕ_μ.

However, if the proper motion is measured (which is necessary in any case to determine the distance through eq. [6]), one already knows ϕ_μ. From equation (10), the three vectors ∇v_r, $\boldsymbol{V}_\perp$, and $\boldsymbol{\Omega}_\times d_{\rm LMC}$ form a triangle, so by the law of sines,

$$V_\perp = \frac{\sin(\phi_\nabla - \phi_{\rm ph})}{\sin(\phi_\mu - \phi_{\rm ph})} |\nabla v_r| \,, \quad (11)$$

where ϕ_∇ is the observed position angle of the kinematic line of nodes. The quantities on the right-hand side of equation (11) are all observables. Assuming that the errors in the measurements of $\boldsymbol{\mu}$ and ∇v_r are isotropic, so that $\sigma(\phi_\mu) = \sigma_\mu/\mu$ and $\sigma(\phi_\nabla) = \sigma_\nabla/|\nabla v_r|$, one can evaluate the error in $d_{\rm LMC} = V_\perp/\mu$ by taking the derivatives of equation (11) with respect to the various parameters. I find

$$\left(\frac{\sigma_d}{d}\right)^2 = \csc^2(\phi_\mu - \phi_{\rm ph}) \left[\left(\frac{\sigma_\nabla}{V_\perp}\right)^2 + \left(\frac{\sigma_\mu}{\mu}\right)^2\right] + \left[\frac{\sin(\phi_\mu - \phi_\nabla)}{\sin(\phi_\nabla - \phi_{\rm ph}) \sin(\phi_\mu - \phi_{\rm ph})}\right]^2 \sigma_{\rm ph}^2 \,, \quad (12)$$

where $\sigma_{\rm ph}$ is the error in the determination of $\phi_{\rm ph}$.

Equation (12) differs from its naive relative, equation (6), in two ways. First, the entire error in equation (6) is now multiplied by a factor $\csc(\phi_\mu - \phi_{\rm ph})$. Second, there is a new term which is related to the uncertainty in the photometric position angle. To understand the importance of these changes, I first introduce representative values of the parameters. I choose $\phi_\mu = 97^\circ$ from the proper-motion measurement of Jones et al. (1994), $\Omega_\times = 25$ km s^{-1} kpc^{-1}, and $V_\perp = 325$ km s^{-1}. Together, these imply $\phi_\nabla = 14^\circ$, thus $(\phi_\mu - \phi_{\rm ph}) = 107^\circ$, $(\phi_\nabla - \phi_{\rm ph}) = 15^\circ$, and $(\phi_\mu - \phi_\nabla) = 92^\circ$. The fact that ϕ_μ and $\phi_{\rm ph}$ are almost at right angles implies that the $\csc^2(\phi_\mu - \phi_{\rm ph})$ term in equation (12) is essentially unity. However, since $\Omega_{\times} d_{\rm LMC} \gg V_{\perp}$, the radial velocity gradient due to $\boldsymbol{V}_{\perp}$ is a relatively minor perturbation on the gradient due to internal motion, and so ϕ_{∇} is not much different from $\phi_{\rm ph}$. Hence, the factor $\sin(\phi_{\nabla} - \phi_{\rm ph}) = 0.26$ in the denominator of the last term is relatively small. This means that $\phi_{\rm ph}$ must be measured quite accurately if one wants a precise measurement of the effect of the transverse velocity. Specifically, the last term in equation (12) is $(3.7\sigma_{\rm ph})^2$. At distances from the center $\gtrsim 2\overset{\circ}{.}5$, the rotation curve tends to flatten, and so $\boldsymbol{V}_{\perp}$ becomes a larger relative perturbation causing $(\phi_{\nabla} - \phi_{\rm ph})$ to grow and thus making the measurement somewhat easier. Nevertheless, imprecise knowledge of $\phi_{\rm ph}$ is likely to be a major limitation of the method.

4. MEASUREMENT OF $\phi_{\rm ph}$

To achieve 2% precision in σ_d/d (which generally seems feasible from the standpoint of the ∇v_r and $\boldsymbol{\mu}$ measurements) would require measuring the position angle to $\sigma_{\rm ph} \sim 0\overset{\circ}{.}3$, or 0.005 radians. It is difficult to believe that this can be achieved using surface photometry alone. Recall that one is not actually interested in the best fit to the major axis of the isophotes. Rather, one wants to know the position angle of the line that crosses the plane of the sky. Certainly star formation, dust, etc., corrupt the surface brightness profile too much to extract information at this level of precision. It should be possible to make a more accurate assessment of $\phi_{\rm ph}$ using star counts particularly of clump giants. Using the method of Gould (1995a), one may show that this technique can determine $\phi_{\rm ph}$ with precision

$$\sigma_{\rm ph} = \left[\frac{N_{\rm cg}}{8}\left\langle\left(\frac{d\ln F}{d\ln R}\right)^2\right\rangle\right]^{-1/2}\frac{\cos i}{\sin^2 i} \sim 0\overset{\circ}{.}2\left(\frac{N_{\rm cg}}{10^6}\right)^{-1/2}, \tag{13}$$

where $N_{\rm cg}$ is the number of clump giants, i is the inclination of the disk, and $F(R)$ is the (assumed axially symmetric) radial profile of the LMC disk, and where I have assumed $i = 30°$ and $\langle(d\ln F/d\ln R)^2\rangle = 6$, which is valid for an exponential disk.

However, clump giants provide another, independent route to the measurement of the position angle. Clump giants have a dispersion in the I band of only $\sigma_{\rm cg} = 0.15$ mag (Udalski et al. 1998). The stars on the near side should therefore be brighter than those on the far side by a significant fraction of this dispersion. Averaging this effect over the whole disk, I find that $\sigma_{\rm ph}$ can be determined to a precision

$$\sigma_{\rm ph} = \left(\frac{N_{\rm cg}}{2}\right)^{-1/2}\frac{\ln 10}{5}\frac{d_{\rm LMC}}{\langle R^2\rangle^{1/2}}(\csc i)\sigma_{\rm cg} \sim 0\overset{\circ}{.}15\left(\frac{N_{\rm cg}}{10^6}\right)^{-1/2}, \tag{14}$$

where I have assumed an exponential scale length of $\alpha = 1\overset{\circ}{.}7$ (Feitzinger et al. 1977), so that $\langle R^2\rangle/d_{\rm LMC}^2 = 6\alpha^2$. The challenges to actually carrying out such a measurement would be formidable. Just maintaining a constant photometric zero point at the level of $\sigma_{\rm cg}/N_{\rm cg}^{1/2} \sim 10^{-4}$ mag over fields separated by $\sim 10°$ would be difficult. In addition, one would have to correct for differential reddening, probably from the clump giant colors, but to do so would require an accurate estimate of $E(V-I)/A_I$. This could be made empirically by looking at the correlation between $V-I$ and I at fixed position but might not be easy.

In principle, it is also possible to measure $\phi_{\rm ph}$ from the transverse velocity field measured from the proper motions. In practice, however, the errors in this determination are too large for it to be useful. Note that the internal transverse motions do not increase the uncertainty in $\boldsymbol{\mu}$. The uncertainty in the transverse velocity field at any particular point is much smaller than either the dispersion or the measurement error, and there is no uncertainty in the mean internal motion averaged over the whole population: the mean internal motion is zero.

5. DISCUSSION

I have outlined how the radial velocity gradient method could be applied to measure $d_{\rm LMC}$ with statistical errors of 2% or less. Of course, as in most distance measurements beyond the solar neighborhood, the largest potential source of errors is systematics. Examples of effects that would generate such systematic errors are noncircular motions and/or warps in the LMC disk and contamination by material along the line of sight. For example, Weinberg (1999) has recently shown that resonant interactions between the Milky Way and the LMC can profoundly disturb the LMC disk.

However, given the mass of data required to make the measurements, it should be possible to conduct many tests for systematics. For example, noncircular motions would affect both the radial velocity gradient and the orientation of the photometric line of nodes. The latter would have a larger impact on the distance simply because its coefficient in equation (12) is ~ 3.5 times larger. Such motions should be revealed in the comparison of the clump giant star count and photometric methods for measuring $\phi_{\rm ph}$: the star-count method would be affected by noncircular motions, while the photometric method would not. Both warps and noncircular motions could be tested by comparing the radial velocity field with the transverse velocity field obtained from proper motions. Similarly, it is possible to search the radial velocities for evidence of unassociated material along the LMC line of sight (Graff et al. 1999).

While an all-out search for systematic effects probably requires the full data set, substantial initial investigations can be made with existing photometric catalogs or with radial velocity studies now underway (e.g., Suntzeff, Schommer, & Hardy 1999).

I have estimated that *FAME* will obtain 8600 proper motions with a mean precision of 250 μas yr^{-1}. If *FAME* is not launched, what are the prospects for matching this performance? Clearly *GAIA*, which is also a survey mission but with much higher precision and fainter magnitude limits, could easily meet this standard. However, given its later launch data and larger analysis time, *GAIA* would require an additional decade to produce results. *SIM* certainly has the capability to make these measurements, but whether it would make so extensive a survey is open to question. Recall that the proper-motion measurements need only be a factor of a few better than the internal dispersion (~ 150 μas yr^{-1}). For $V \sim 15$ stars, *SIM* could do an order of magnitude better than this in 1 minute. Allowing another minute for pointing and assuming a total of four position measurements per star, 8600 proper-motion measurements would require about 1000 hr. From equation (8) only ~ 100 stars would be needed to measure $d_{\rm LMC}$ to $\sim 1\%$, and this could be done in only about 10 hr. In this case, however, one would lose much of the ability to check for systematics from a comparison of the radial velocity and proper-motion fields. In brief, *FAME* is the instrument of choice to make the proper-motion measurement.

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