Title:	Are Dead Wires Dangerous?
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Issue Date:	Feb-1929
Publisher:	Ohio State University, College of Engineering
Citation:	Ohio State Engineer, vol. 12, no. 4 (February, 1929), 6-7, 24.
URI:	http://hdl.handle.net/1811/34553
Appears in Collections:	Ohio State Engineer: Volume 12, no. 4 (February, 1929)

ARE DEAD WIRES DANGEROUS?

By PROF. A. F. PUCHSTEIN AND K. Y. TANG

Whenever a high voltage circuit, as a high-tension power line, is in close proximity to a dead wire, particularly when they are parallel to each other for a considerable distance, a potential difference is induced between the wire and ground. This potential is due to the presence of the magnetic and the electric fields set up by the power line. The magnetic factor is usually of minor importance and its principles are well understood. Attention will, therefore, be confined to the more important but less easily understood electrostatic action.

To explain this action, consider two metallic wires, 1 and 2, strung above the earth parallel to each other as in Fig. 1. With a battery connected between wire 1 and earth, wire 1 will assume a potential, say V, above ground equal to the battery voltage.

The conductor 2 will assume a potential above ground intermediate between V and zero, even though it be isolated. The potential of 2 above that of the ground will be higher the closer it is to 1 and lower the closer it is to earth.

This action is easily explained from Fig. 2 (a) where the curved lines represent lines of stress, which assume the form of arcs of circles with centers on the earth plane, C'C. Here, the full difference of potential V will exist between 1 and the points a, b, c, etc.

The voltage drop per inch along any such line, as 1-b, will be lower for long than for short ones. This voltage drop will be affected only very slightly if a second isolated conductor, 2, be added. The presence of this added conductor has the effect of shortening the thickness of insulation (air) between 1 and c and changes slightly the distribution of the lines of stress. As the area of path occupied by a line of flux is not uniform, being large at points near the earth and small in the vicinity of conductor 1, the voltage drop per inch length is greater at points close to conductor 1 than at those farther away. Since the space surrounding the conductors is occupied by air, which is an insulator, the arrangement will have the properties of an electric condenser.

The calculation of the voltage between any two points in such a system may be simplified by considering the earth as replaced by conductor 1', Fig. 2(b), called the image of 1, equal with 1 and as far below the earth's surface as 1 is above it. All the lines 1-a, 1-b, etc. will be unchanged. Hence, the system Fig. 2(a) may be replaced by the new system Fig. 2(b).

When the battery is connected as shown, conductor 1 takes a charge, +q, and conductor 1' a charge, —q. These charges are proportional to the voltage and their values are found as shown later. Under this condition, the isolated conductor, 2, takes no charge because the lines of stress which enter it do not end there, but pass on through in a transverse direction. This is no longer true if conductor 2 is connected to ground by a metallic wire, in which case, conductor 2 takes on a charge, but its potential drops to zero.

The problem of finding the potential assumed by conductor 2 can be solved by establishing a relation which is suitable to be used as a measure of the potential difference between any two conductors. Such a relation may be obtained by taking a pair of spheres, A and B, and connecting them to a battery as in Fig. 3(a). Sphere A will take on a charge +q, and B a charge —q. These charges can be measured by introducing the concept of a unit charge, which has a value such that it will exert a force of one dyne

$$(1 \text{ dyne} = \frac{1}{444000} \text{lb.})$$

on a similar charge at a distance of one centimeter. This value is called an electrostatic unit of charge. The force between two charges in electrostatic units is given by

$$f = \frac{q_1 \; q_2}{r^2} \; dynes \qquad (1)$$

where q_1 and q_2 are the charges, r the distance between them in centimeters. If one of the charges be made unity, the expression for the force becomes

$$f = \frac{q}{r^2} \qquad (2)$$

Thus, the voltage of any body could be found by bringing such a unit charge to within, say, 1 cm. of the charged body and measuring the force upon it. Another way is to take the integral

$$W = \int_{\infty}^{r_1} f \, dr = \int_{\infty}^{r_1} \frac{q}{r^2} \, dr \qquad (3)$$

$$= \frac{q}{r_1} \qquad (4)$$

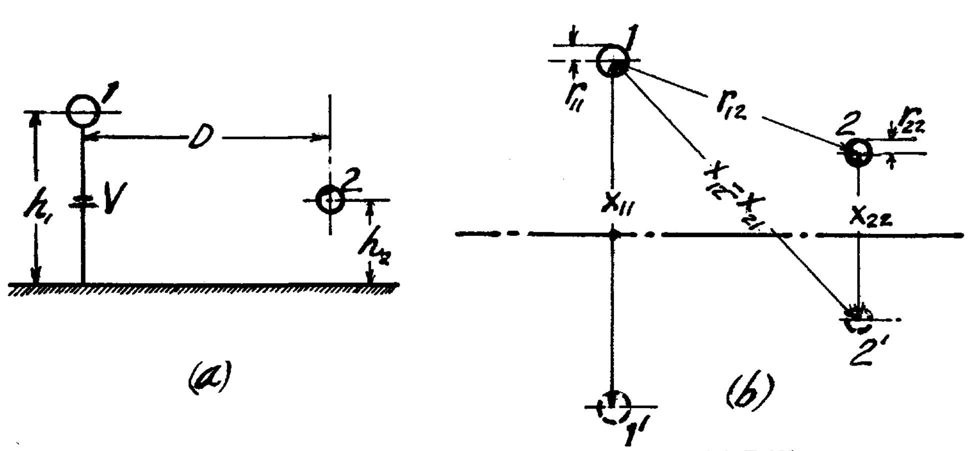

FIG. 1—ARRANGEMENT OF CONDUCTORS AND NOTATION

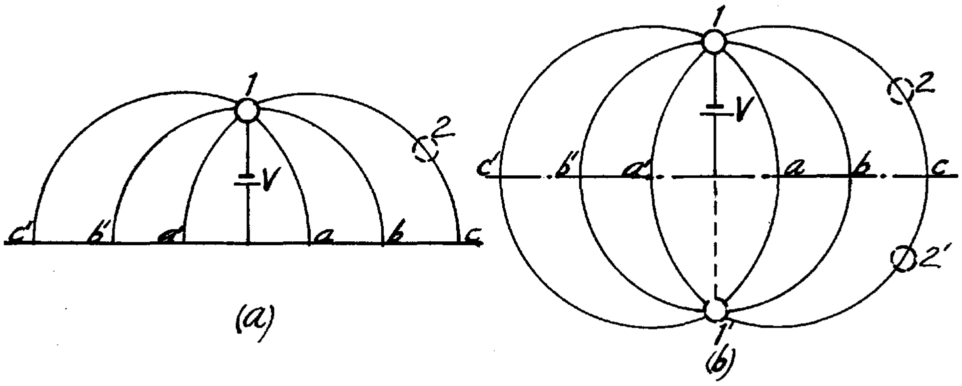

FIG. 2—FIELD OF A SINGLE WIRE TO EARTH

which represents the work done in bringing a unit charge from infinity to the point r_1. This work is independent of the path followed. Any other point at which the potential is zero may be used for a limit in place of infinity. Since q is proportional to the potential V, then from (4) W is proportional to V and may be used as a measure of V. The use of W has certain advantages over f and it has been used almost exclusively for investigation of this kind.

If a unit charge be moved from A to B or vice versa, the work represented is

$$W = 2\int_d^{r_1} \frac{q}{r^2}\, dr \qquad (5)$$

$$= 2\left(\frac{q}{r_1} - \frac{q}{d}\right) \textit{ dyne-centimeters} \qquad (6)$$

From this point view, potential is related to work in such a way that

$$W = CV \qquad (7)$$

where C is a constant. When electrostatic units are used, C is unity, and,

$$W = V \qquad (8)$$

thus numerically, the work in dyne-cm. required to move the unit charge from A to B is equal to the potential.

For the case of parallel wires, the unit charge is used as before to find the relation between work and potential. Consider two parallel wires as in Fig. 3(b). Let q be the charge in electrostatic units per centimeter length of conductor 2 and r the radius of the conductors; then the force produced on a unit charge at any point P by the charge on an element dx of conductor at x is

$$df = \frac{q\,dx}{x^2 + y^2} \qquad (9)$$

The horizontal component of this force will be cancelled by that of an equal charge, qdx, at —x. Therefore, only the vertical component need be considered and its value is

$$\frac{qdx}{x^2+y^2} \cdot \frac{y}{\sqrt{x^2+y^2}} = \frac{qydx}{(x^2+y^2)^{3/2}} \textit{ dynes} \qquad (10)$$

Assuming that the length is large compared with the distance between wires, the force due to the whole charge on conductor 2 will approach

$$\int_{-\infty}^{+\infty} \frac{qydx}{(x^2+y^2)^{3/2}} = \frac{2q}{y} \textit{ dynes} \qquad (11)$$

The work in moving the unit charge from conductor 2 to conductor 1 is

$$W = \int_r^d \frac{2q}{y} dy = 2q \textit{ loge} \frac{d}{r}$$

If conductor 1 also carries a charge —q, the work done will be

$$2W = 4\, q \textit{ loge} \frac{d}{r} \qquad (13)$$

The expression (12) is called a potential coefficient, and is usually represented by P with appropriate subscripts, as Pmn.

Refer to Fig 1(b). If the potential between 1 and its image 1′ is V_1 with a charge of q_1 per unit length, and if the potential between conductor 2 and its image is V_2 with a charge q_2 per unit length, the work required to move a unit charge from 1′ to 1 will be

$$W_1 = V_1 = 2q_1 \textit{ loge} \frac{x_{11}}{r_{11}} + 2q_2 \textit{ loge} \frac{x_{12}}{r_{12}}$$
$$= P_1\, q_{11} + P_{12}\, q_2 \qquad (14)$$

from 2′ to 2 it is

$$W_2 = V_2 = 2q_1 \textit{ loge} \frac{x_{12}}{r_{12}} + 2q_2 \textit{ loge} \frac{x_{22}}{r_{22}}$$
$$= P_{12}\, q_1 + P_{22}\, q_2 \qquad (15)$$

where 1′ and 2′ are at zero potential. The notation will be clear from Fig. 1(b).

It is now easy to generalize for a larger number of conductors, and the general equations first given by Maxwell may be written by inspection

$$V_1 = P_{11}q_1 + P_{21}q_2 + P_{31}q_3 + \ldots + P_{n1}q_n$$
$$V_2 = P_{12}q_1 + P_{22}q_2 + P_{33}q_3 + \ldots + P_{n2}q_n \qquad (16)$$
$$V_n = P_{1n}q_1 + P_{2n}q_2 + P_{32}q_3 + \ldots + P_{nn}q_n$$

where Pmn = Pnm. In order to be solvable, such a system of equations must contain as many equations as conductors and an equal number of unknowns, either V's or q's or both. When any wire is isolated its charge q will be zero, and when connected to ground its potential V will be zero.

In the case where all the wires are isolated, an additional relation, $q_1 + q_2 + \ldots + q_n = 0$, is needed. The mode of solution is to solve for the unknown q's in the equations with the known V's, and to substitute these calculated q's in the equations containing the unknown V's. For practical units (volts, coulombs per mile, and common

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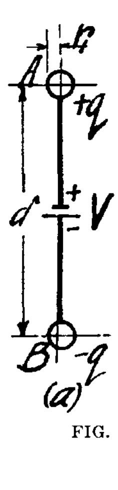

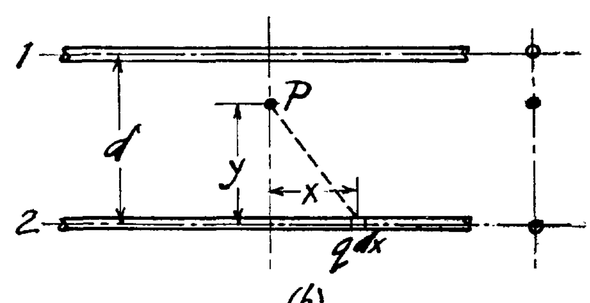

FIG. 3—FORCE ON A UNIT CHARGE BETWEEN CHARGED BODIES

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logarithms) the value for C in (7) is $\frac{3.883}{10^8}$ Also,

1 electrostatic volt = 300 volts
1 electrostatic ampere = $3\frac{1}{3}x^{-10}$ amperes
1 mile = 161,000 centimeters.

Numerical Example

In Fig. 1, let $V_1 = 50{,}000$ volts, and
$h_1 = 480$ inches; $h_2 = 240$ inches; D = 240 inches

$r_{11} = .162$ inches $\quad x_{11} = 960$ inches
$r_{22} = .051$ inches $\quad x_{22} = 480$ inches
$r_{12} = 340$ inches $\quad x_{12} = 760$ inches

$$P_{11} = 4.6\ log_{10} \frac{960}{.162} = 17.35$$

$$P_{12} = 4.6\ log_{10} \frac{760}{340} = 1.60$$

then Maxwell's equations are from (16)

$$V_1 = P_{11}q_1 + P_{12}q_2$$
$$and\ V_2 = P_{12}q_1 + P_{22}q_2$$

Since conductor 2 is isolated, its charge q_2 is zero and the equations become

$$V_1 = P_{11}q_1\ ;\ 50{,}000 = 17.35\ q_1$$
$$and\ V_2 = P_{12}q_1\ ;\ V_2 = 1.6\ q_1$$

Solving

$$q_1 = \frac{50{,}000}{17.35} = 2880$$

and

$$V_2 = 1.6 \times 2880 = 4610\ Volts$$

Except for short lengths, this voltage, V_2, is independent of the length of wire, but the danger to life increases with the length because the quantity of electricity which may be discharged through the body is directly proportional to the length.

A case where trouble of this type may be expected is that of a telephone line running parallel to a power line, even if located on opposite sides of a highway.