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THE RELATIVE INTENSITY OF THE HARMONICS OF A LECHER SYSTEM. (EXPERIMENTAL).

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In their fourth paper* on the free vibrations of a Lecher system, Blake and Sheard have shown how the tone intensity depends very much upon the edge-on distance between the plates, but they were unable on account of oscillator deterioration among other things to summarize this relation. We have made a careful study of the various factors that help to determine the tone intensity and present the results of our study in this paper.

I. APPARATUS.

The apparatus used was that employed by Blake and Sheard except that the brass plates used were 1 mm. thick instead of 4.5 mm. It is shown diagrammatically in Figure 1, where the letters are the same as in Figure 1 of Blake and Sheard's paper IV. This necessitated making eight small holes in pairs through the plates around the circumference through which the strings were passed and knotted. The diameter of each hole was about 0.6 mm. while that of the plates was 5.0 cm. The total area of the holes was thus about 0.4 per cent of the area of the plate,

* Physical Review, N. S., IX, p. 177, 1917.

hence the effect of these holes could be entirely neglected, particularly since the coupling was always very loose, viz., 11 cm. On account of the tension of the strings and the thinness of the plates, the plates were slightly bowed, that is convex toward each other. This effect was never greater than 8 parts in 1100, hence could be neglected.

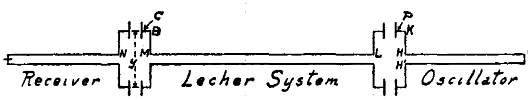

Figure 1.

II. Variable Factors.

The following variable factors required study. First, position of the bridge for each tone; second, influence of the spark-gap length; third, length of the oscillator; fourth, edge-on distance, y, between the plates; fifth, change of coupling, that is, change of the face-on distance, x, between plates; sixth, oscillator deterioration, that is, irregularity due to deposit of soot upon the metal rods of the spark-gap in oil; seventh, possible change in the sensitivity of the thermocouple even though soldered. While any one variable was being investigated, if another variable suffered a change it constituted a source of error for the time being.

Since the tension on the three circuits, oscillator, Lecher and receiver circuits, was kept always uniform, small variations of the lengths of the circuits could either be neglected or taken into account as needed. Again as heretofore the receiver circuit was always just half of the Lecher circuit. The fifth variable above, x, has been studied, but its importance demands a special paper.

Our method of handling the first two variables was as follows: With a given x and y, and a given oscillator length, having set the spark-gap at about the position for maximum intensity for a given tone, a bridge curve was taken across a given peak forward and backward. Then the bridge being placed at the peak of a curve, a spark-gap curve was taken. Without stopping to plot the observations at the time, the correct position of the bridge could be readily told to well within 0.5 mm. by mere inspection of the observations.

A sample of such curves is shown in Figure 2. For the bridge at 150 the lower of the two curves of Figure 2 was taken, then a bridge curve was taken and the reading of the maximum, viz., 100, was so far above the maximum of Figure 2 (lower curve) viz., 66, that another spark-gap curve was taken, the upper curve of Figure 2. These two curves illustrate very clearly and forcibly the meaning of oscillator deterioration. Evidently with so narrow a gap, viz., 0.00016 inch, it is easy for carbon soot from the decomposed oil to bridge, partially or wholly, the gap and thus decrease the intensity materially.

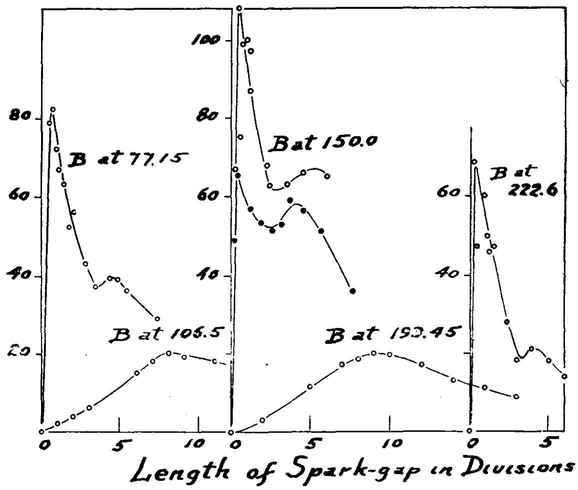

Figure 2.

The matter would be easy to handle if the gap were occasionally wholly bridged in this way, for observations under such conditions could easily be detected, and hence eliminated. If, however, in spite of the fact that we used only filtered oil the gap was only partially bridged by the carbon soot there does not appear any easy way to correct for this. When for the sake of inspection the oscillator rods were removed, the ends that constituted the spark-gap would be covered in places with dark spots, due to extremely fine soot being deposited in an extremely thin layer. This matter of oscillator deterioration would not be serious provided one could always employ a check receiver to advantage. This was indeed done in a considerable part of the work here reported. But we shall show later on in this paper that the profitable use of a check receiver is very limited.

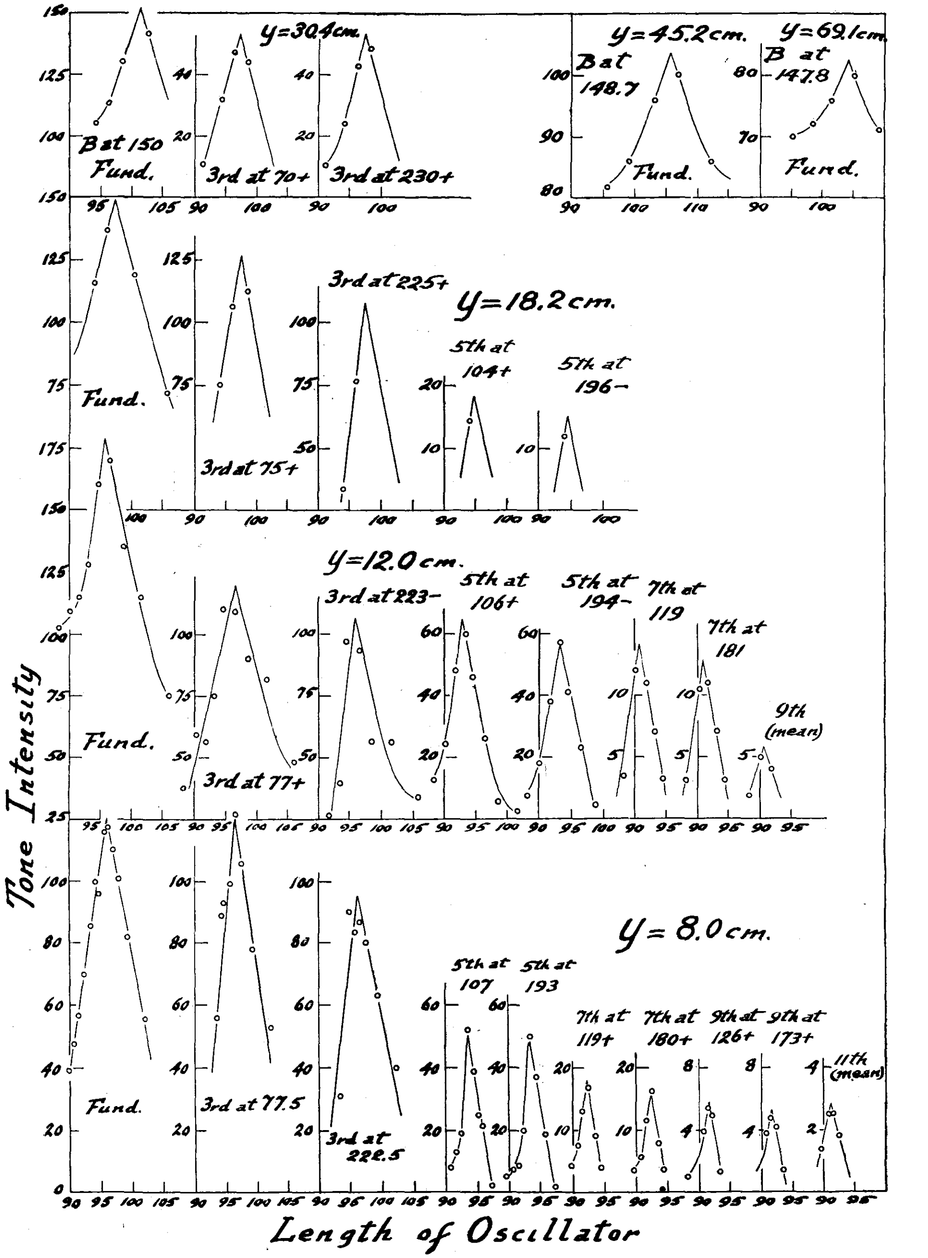
y=30.4cm.
B at 150
Fund.
3rd at 70+
3rd at 230+
y=45.2cm.
B at 148.7
Fund.
y=69.1cm.
B at 147.8
Fund.
y=18.2cm.
Fund.
3rd at 75+
3rd at 225+
5th at 104+
5th at 196−
y=12.0cm.
Fund.
3rd at 77+
3rd at 223−
5th at 106+
5th at 194−
7th at 119
7th at 181
9th (mean)
y=8.0cm.
Fund.
3rd at 77.5
3rd at 222.5
5th at 107
5th at 193
7th at 119+
7th at 180+
9th at 126+
9th at 173+
11th (mean)
Tone Intensity
Length of Oscillator

Figure 3.

Neglecting momentarily the influence of oscillator deterioration, the only way in which the third and fourth variables could be studied was to get for every new length of oscillator both bridge and spark-gap curves for every peak; all this for a given y. And then for a new value of y we had to repeat all the above operations. Table I shows the various values of y used. It also shows the longest and shortest oscillator lengths, the length of the Lecher wires and of the receiver. As heretofore these lengths are the straight-away lengths, GH, LM, NT, Figure I.

III. Summary Curves and Tables for Variable y.

Proceeding in the manner indicated above and plotting all bridge and spark-gap curves we obtained summarizing curves of intensity such as we have shown in Figure 3. They were obtained by tabulating the peaks of the spark-gap curves for different oscillator lengths.

It is plain from an inspection of Figure 3 that the higher tones are proportionately greater the smaller the y is, a thing Blake and Sheard found. For the curves for the values of y shown in the figure up to and including 30.4 cm., the fundamental was taken with the bridge at 150. But for values of y greater than 15 cm. the fundamental splits up into two peaks as the curves of Figure 4 clearly show (see also Curve II, Figure 5). Consequently in Figure 3 for $y=18.2$ cm. and 30.4 cm., the relative intensity of the fundamental is too low. A further study of Figure 3 shows also that as y increases the length of oscillator for maximum resonance increases. Our results on this matter are summarized in Curves III, Figure 5. For harmonics above the third the intensity is so low for the larger values of y that one is not able to examine how the optimum oscillator length varies. For the fifths the optimum oscillator length for $y=18$ is unquestionably somewhat longer than for $y=12$, but the variation is proportionately less than for the thirds and fundamental.

IV. Various Corrections.

The sets of curves for Figure 3 for different values of y cannot be directly compared with one another for several reasons. First, those of $y=8$ cm. (together with the fourth and sixth sets shown in Table I) were taken with a different thermo-couple from the rest. Second, the different sets were taken often several days apart and the oscillator deterioration as well as possible change in sensitivity of the thermocouple needed to be eliminated or corrected for. Third, the change in room temperature, while not great, undoubtedly affected the dielectric constant of the olive oil used in the spark-gap and consequently the energy as read at the receiver.

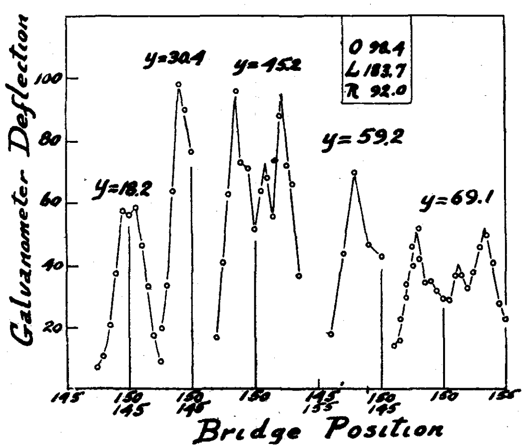

Figure 4.

In the light of our experiments there seems to be a distinct deterioration in the sensitivity of our thermocouple even though soldered. For with a polished spark-gap and other conditions the same the intensity was often distinctly less for all the tones of a set for a given y than when the same set was taken earlier. Any figures we could give in illustration of this fact would not be worth much, for manifestly the other factors, such as oscillator deterioration and temperature change, are not readily differentiated from the sensitivity changes of the thermocouple. The fact that these three factors cannot be readily differentiated one from another shows, too, the futility, for the object we had in mind, of employing a check receiver. Moreover, while for the bridge curves the check receiver helped to eliminate oscillator variation, it proved wholly useless for spark-gap curves. The check receiver was placed, after Blake and Sheard, near the oscillator spark-gap, but in general the maximum for the two receivers did not occur at nearly the same spark-gap length, hence neither curve could be used to correct the other except by an approximation method of some sort. Hence we abandoned its further use. Accordingly, the only safe way to make a direct comparison of the curves of Figure 3 was to get some one tone, say the fundamental, to a common basis.

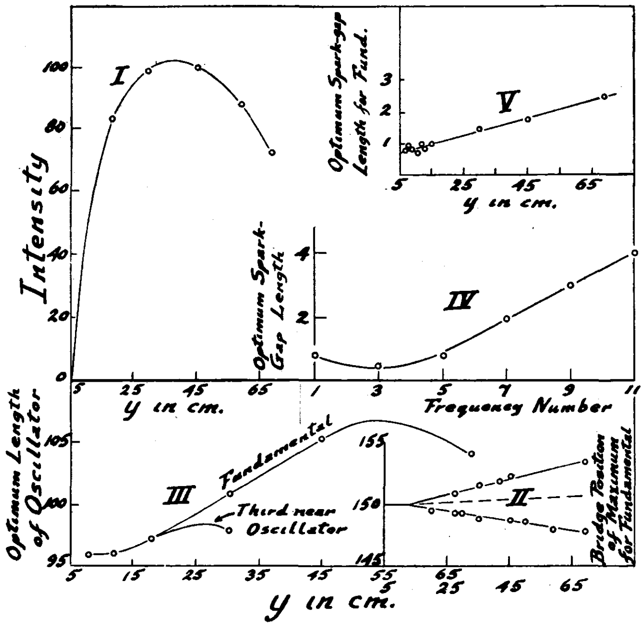

Figure 5.

Before describing the manner of reducing our corrections to a common basis it is well to point out first, however, that the optimum spark-gap length varies with the frequency-number just as Blake and Sheard found (see their Figure 12). Curve IV of Figure 5 summarizes roughly this relation as the mean value for all the y's used. The optimum spark-gap length for a given tone is not independent of y, however, as Curve V of that figure shows for the fundamental. It is clear from Curve IV, that the optimum spark-gap length decreases with the frequency numbers in general, reaches a minimum for the third harmonic, and rises again for the fundamental. Curve V shows that the optimum spark-gap length for the fundamental increases with increasing y in a linear ratio. This is probably true for the higher tones also, though the variation is so little outside of the limits of experimental error that it would be difficult to show this for the higher tones. Curves IV and V show the absolute need for taking spark-gap curves if the tone intensities are to be compared.

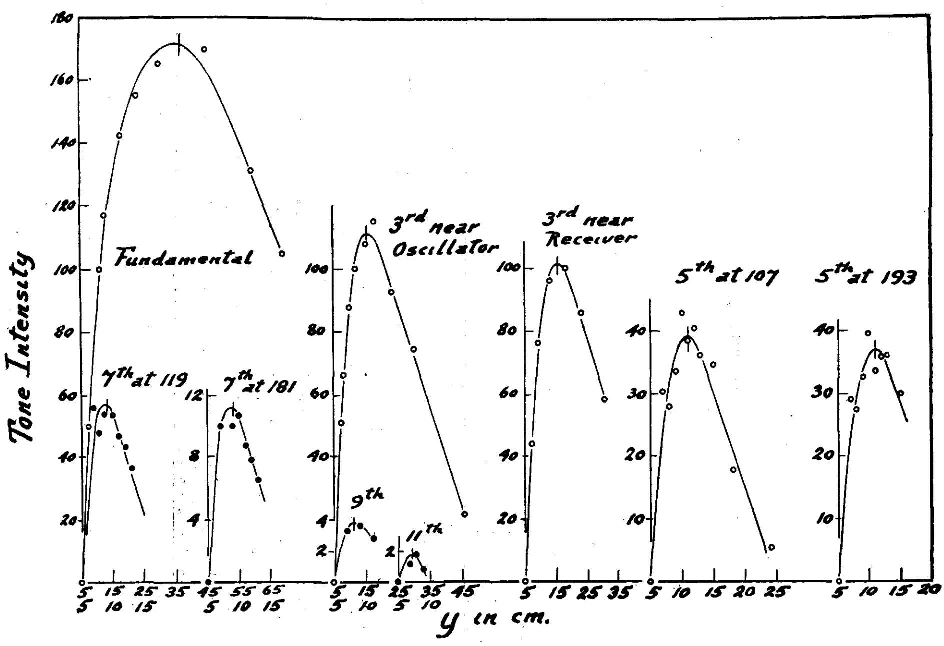

Figure 6.

To eliminate the various curves discussed above and to get at a common basis for comparison of the various sets of curves for the different values of y we proceeded as follows. Starting with a given oscillator length GH, and a given total length GT of the system, Figure 1, the value of y was changed from large to small values by steps in as short a time as possible. For each y we located the fundamental peak or peaks and then took spark-gap curves. In this way Curve I of Figure 5 was obtained. The curve is extrapolated to zero for $y = 5$ cm., for at this distance the inner edges of the plates would touch. We then reduced all the summarizing sets of Figure 3, together with all the other sets not shown, to correspond to the standard for the fundamental for $y = 18.2$, viz., 142 (see Figure 3). In this way the curves of Figure 6 were obtained. They certainly cannot be much in error.

The curves for the ninth and eleventh harmonics represent an average set of values since not every peak was investigated for those tones for every change of oscillator. The other curves of Figure 6 give the fundamental and the thirds, fifths and sevenths on both sides of the fundamental. By taking the average of the two thirds, the two fifths and the two sevenths, the damping is eliminated and the results can be compared directly with the fundamental.

V. The Question of Damping.

Before making the comparison it is well to ask two questions. How is the intensity of a peak affected by the fact that it is split in two? Is the damping sufficiently small that by taking the arithmetical mean of any two corresponding tones equally distant from the fundamental it can be eliminated? We proceed to the answers to these questions.

For the results shown in this paper the coupling, x, was kept constant at 11 cm. But we have shown before that for $x = 8$ cm. and less and for small values of y the fundamental is split up. Evidently, then, an increase of y acts like a decrease of x; it causes two periods for a given tone. In our arrangement we detect these two possible periods (or wave lengths) by means of a sliding bridge. Manifestly, when one period is present, the other is absent, so far as the receiver is concerned. In other words, *all* of the energy is in *each* of the two peaks of the fundamental.

The second question, it would seem, can be answered in this way. The fact that the peak on the oscillator side of the fundamental is higher than the corresponding peak on the receiver side of the fundamental makes it plausible to say that if I_0 is the intensity at the oscillator spark-gap say and I the intensity at some point along the system the equation $I = I_0 e^{-ax}$ would hold where a is the logarithmic decrement.

Whence for two different distances x_1 and x_2 we can say $I_1 = I_o e^{-ax_1}$ and $I_2 = I_o e^{-ax_2}$ whence

$$a = \frac{\log_e I_1/I_2}{x_2 - x_1}$$

Now for the third harmonic $x_2 = \frac{4\lambda_3}{2}$ and $x_1 = \frac{2\lambda_3}{2}$ whence $x_2 - x_1 = \lambda_3$. Similarly for the fifths and sevenths $x_2 - x_1$ would equal λ_5 and λ_7 respectively. The ratio $\frac{I_1}{I_2}$ for the thirds is $\frac{111}{101} = 1.099$. For the fifths and sevenths this ratio is $\frac{38.5}{36.9} = 1.044$ and $\frac{11.4}{11.2} = 1.017$ respectively.

Using the figures for λ_s given for $y = 8$ cm. viz., $\lambda_3 = 145$, $\lambda_5 = 86$, $\lambda_7 = 61$, we get a to be 0.00065, 0.00050, 0.00028 respectively. Thus the value of a apparently decreases for the higher harmonics. It may fairly be questioned, however, whether the question of damping can be treated as simply as here given. Using the values of λ_s corresponding to the *optimum* values of y, which we shall call y_s, viz., $\lambda_3 = 150$, $\lambda_5 = 87$ and $\lambda_7 = 61$, changes the values of a but little. Now Blake and Sheard* have shown that the electrostatic leakage, which they called "θ (y)" was greater for the lower tones than for the higher tones. This would readily explain the larger values of a for the lower tones.

The important thing at present, however, is that in any case the value of a must be very small. Even though we measured x, not from the oscillator spark-gap, but from the Lecher plates, the value of a would be doubled only. There seems no good justification for doing this, however.

For the third harmonic, using the value of a first given, 0.00065, and calculating the intensity I of the third at the middle of the Lecher system we get $I = 105.9$. The arithmetic mean of 111 and 101 is 106.0. This is the justification for taking the arithmetic mean of the two tones either side of the fundamental to get the tone intensity of any tone that can fairly be compared with the fundamental. The question of damping is thereby eliminated.

* Physical Review, l. c.

It seems very likely that the question of damping can be handled thus simply for the cases here considered. Blake and Sheard have shown that we are dealing here, when the coupling is loose ($x=11$ cm. is a very loose coupling), with the free vibrations strictly and since our system is a strictly non-radiant system the damping must be very small. It is perhaps an open question whether the logarithmic decrement thus measured was the decrement of the receiver merely or of the total oscillating system. Likely the former, however, since the oscillator and the distance y was purposely changed so as to have the energy at the receiver the maximum possible for each tone, with a given constant input of energy at the oscillator spark-gap.

VI. Relation Between Tone-Intensity and Edge-on Distance.

Thus the energy at the receiver is distributed among the various tones as follows, the figures being read off directly from the curves of Figure 6: fundamental 172, third 106, fifth 37.7, seventh 11.3, ninth 3.8, eleventh 1.8. Expressed in per cent. the figures run 100%, 61.63%, 21.92%, 6.57%, 2.21%, 1.05%. With the more complete data given in this paper we have thus been able to determine quantitatively the influence of the edge-on distance between plates upon the tone-intensity. Blake and Sheard were able to give a qualitative answer only.

If we plot the optimum distance between plates, y_s, against the frequency number, we get Curve III of Figure 9. With an average error of less than 1 per cent. and a maximum error of 1.7 per cent. this curve gives the empirical relation

$$\sqrt[4]{s}\ \log_e y_s = \text{Constant.}$$

We are not prepared at present to state whether such a relation has any important significance. Calculating the distances y_s required to fit the formula we get the following values: 36.8, 15.5, 11.1, 9.15, 8.00, 7.25. In the curves of Figure 6 the short lines parallel to the y axis cutting across the curves are shown at these points. Manifestly, within the limits of error, they cut at the top of each maximum.

TABLE I.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
y cm.	Lecher Wires cm.	Re-ceiver cm.	Oscil-lator cm.	MEAN BRIDGE POSITION, B_s						MEAN WAVE LENGTH, λ_s						p	$S\lambda_s$ experi-mental	l_s experi-mental	κ_0/κ exp'i-m't'l	$S\lambda_s$ theor-etical	l_s theor-etical	κ_0/κ theor-etical	% Error in κ_0/κ	% Error in λ_s
				s=1	3	5	7	9	11	s=1	3	5	7	9	11									
						63.88	89.54	80.53	(94.60)								442.90	96.32	14.60	442.90	96.32		0.00	0.00
			103.7		77.05	107.00	119.76	—	—			(86.12)					437.52	23.40	14.64	437.43	23.42		+0.27	+0.02
7.15	180.50	90.2	to	150.00	150.00	150.00	150.00	150.00	150.00	442.90	145.84	86.00	60.44	46.34	37.40	246.32	430.00	10.32	14.58	430.05	10.32	14.60	−0.14	−0.01
			90.1		222.89	193.00	180.20	—	—								423.08	5.66	14.42	423.29	5.62		−1.23	−0.05
						236.12	210.43	219.54	206.10	442.90	145.81	86.01	60.47	46.43	37.58		417.06	3.64	13.71	417.87	3.46		−7.60	−0.24
																	411.40	2.82	11.61	413.38	2.37		−20.6	−0.48
																	440.07	96.47	13.70	440.07	96.47		0.00	0.00
			102.2		77.50	107.08	119.75	126.74	131.05	440.07	145.04	85.82	60.42	46.46	37.67		435.12	23.95	13.62	435.42	23.90		−0.58	−0.07
8.0	179.95	89.9	to	150.00												246.47	429.10	10.65	13.82	428.90	10.69	13.70	+0.88	+0.05
			88.2							440.07	145.14	85.78	60.41	46.41	37.60		422.94	5.84	13.76	422.87	5.86		+0.44	+0.02
					222.54	192.90	180.17	173.20	168.72								418.14	3.55	14.15	417.69	3.65		+3.28	+0.11
																	414.37	2.30	14.85	413.60	2.47		+8.40	+0.19
						63.66	89.14	79.90									440.36	97.24	13.03	440.36	97.24		0.00	0.00
			102.4		77.28	106.83	119.61	103.00				(86.42)					436.80	24.44	13.15	436.34	24.50		+0.92	+0.08
9.0	180.44	90.2	to	150.09				—	131.31	440.36	145.60	86.36	60.88	46.82	37.94	247.24	431.90	10.88	13.58	430.65	11.11	13.03	+4.22	+0.27
			90.0		222.88	193.19	180.50	173.35	169.25								426.16	5.92	13.84	424.97	6.18		+6.21	+0.28
								196.75		440.36	145.48	86.13	60.71	46.70	37.85		421.38	3.60	14.21	420.30	3.84		+8.30	+0.26
						236.50	210.91	220.36									417.34	2.39	14.48	416.35	2.62		+11.1	+0.24
																	436.38	97.15	12.06	438.72	97.15		−3.91	−0.53
			110.0		77.65	107.00	119.57	126.50		436.38	144.65	86.10	60.87	47.00			433.95	24.75	12.36	434.46	24.74		−1.51	−0.12
10.0	179.40	89.7	to	150.00												247.15	430.50	11.05	13.14	429.20	11.31	12.55	+4.70	+0.30
			86.4		222.30	193.10	180.44	173.50		438.72	144.82	85.84	60.56	46.59			426.09	5.85	14.04	423.92	6.31		+11.9	+0.51
																	423.00	3.15	16.70	419.31	3.97		+33.0	+0.88
												(87.19)					439.28	98.25	11.63	441.31	98.25		−4.98	−0.46
			102.1			62.77	88.37	102.65		439.28	145.86	87.00	61.62	47.40			437.58	25.32	12.08	438.12	25.23		−1.31	−0.12
11.1	180.40	90.2	to		77.00	106.44	119.21	126.34								248.25	435.00	11.25	13.12	433.00	11.65	12.24	+7.18	+0.46
			89.0	149.94	222.86	193.44	180.75	173.70		441.31	146.04	86.60	61.14	47.05			431.34	5.82	14.54	427.98	6.54		+18.8	+0.78
						237.15	211.62	197.44									426.60	3.45	15.33	423.45	4.15		+25.2	+0.74
						62.75						(87.42)					434.75	98.10	10.67	439.63	98.10		−11.1	−1.12
			106.3		77.54	106.72	119.10	126.17		434.75	145.14	87.00	61.95	47.98			435.42	25.53	11.60	436.62	25.33		−3.33	−0.27
12.0	179.20	89.5	to	150.08												248.10	435.00	11.10	13.49	432.20	11.66	12.00	+12.4	+0.65
			88.3		222.68	193.72	181.05	174.15		439.63	145.54	86.44	61.01	46.94			433.65	6.18	17.05	427.07	6.59		+42.1	+1.54
						237.60											431.82	2.14	26.54	420.66	4.22		+121.2	+2.22
																	437.00	99.16	10.16					
			102.1	150.15	76.84	106.17	118.80									249.16	439.80	25.86	11.63			11.79		
13.1	180.20	90.1	to		223.44	194.20	181.50			437.00	146.60	88.03	62.70				440.15	11.13	13.76					
			89.4														438.90	5.11	17.75					
											73.65	44.41	31.62				439.50	100.11	9.81					
			100.6	150.12	76.47	105.71	118.50				74.27	44.66	31.63			250.11	443.76	27.15	10.45			11.50		
15.1	180.10	90.0	to		224.39	194.78	181.75										445.35	11.04	14.37					
			89.1							439.50	147.92	89.07	63.25				442.75	4.71	17.56					
											74.75	45.40					444.65	101.58	9.64					
			105.4	150.15	75.40	104.75					75.15	45.55				251.58	449.70	26.63	11.66			11.18		
18.2	181.04	90.5	to		225.30	195.70											454.75	10.63	16.03					
			94.0							444.65	149.90	90.95												
			102.5	149.15	70.74						79.46													
30.4	180.80	90.4	to	and							79.98					258.13	478.32	28.41	12.30			10.57		

VII. How Does the Capacity of the Condensers Vary With the Edge-on Distance Between Plates and With the Plate Thickness?

Blake and Sheard found that for small values of y the value of κ_0/κ, that is, the end-capacity expressed in equivalent wire-length, was practically constant, as calculated from the data for the different harmonics. This was to be expected on account of the small range of frequencies they worked with, viz., 1 to 9. The range in this paper is but slightly greater, viz., 1 to 11, hence we should expect to confirm their results in this regard. This is indeed the case as Table I clearly shows. Columns 5 to 10 inclusive give the mean positions of all the peaks for all the tones. The upper set of figures in columns 11 to 16 inclusive give the experimental values of λ_s, the wave-length corresponding to the frequency number, s. In column 13 the figures in parentheses are the observed values of the wave-lengths of the fifths obtained by subtracting the two outer fifths from each other and dividing by 2, e. g. $\frac{236.12—63.88}{2}=86.12$. Blake and Sheard observed that the outer internodal spaces for the fifths were distinctly larger than for the inner internodal spaces. Since they did not observe this for the sevenths and ninths within the limits of experimental error they attributed this to the presence of the small ebonite supports, R_1, R_2, R_3, R_4 in the immediate vicinity of these outer peaks for the fifths. Our results confirm theirs. In the calculations the outer fifths were not used. We found, however, that the intensity of the outer fifth near the oscillator is not as great as that of either of the two inner fifths, whereas it should be somewhat greater. This was true for all the values of y for which the outer fifths were measured. Column 17 gives the position C, Figure 1, obtained by adding $3.5+\frac{1}{2}(y—2)$ to the plum-bob reading M as given on the 3-meter stick. It is the reading of the back of the Lecher plates next to the receiver. Column 19 is calculated in the manner indicated in Blake and Sheard's paper (q. v.).* Column 20 is obtained by means of the formula

$$\frac{\kappa_0}{\kappa} = \frac{\lambda_s/2\pi}{\tan\left(\frac{l_s/\lambda_s}{/2\pi}\right)}$$

The first figure in each set in column 18 is obtained by extrapolation as given in Figure 7, full lines, and Figure 8, where the points plotted are the remaining figures in column 18. Naturally there is some degree of latitude in making this extrapolation. For instance, one could so change λ_1, in the sets mentioned in the accompanying note, as to leave the first value in column 20 unchanged from the figure given in the table. We believe, however, that the reader will agree from an inspection of the curves of Figures 7 and 8 that the extrapolated values for λ_1 given in the table are not greatly in error.

Column 23 was obtained in the following way. The first set of column 20 is a decreasing set, the second an increasing set but in each case the first four figures are fairly constant. Using the first four figures of each set and allowing for the slight decrease and increase respectively the first two figures were taken as given in column 23. With these two given, the remaining figures in that column were obtained by graphical methods by assuming that κ_0/κ varied with y according to the relation

$$\kappa_0 \propto \frac{1}{\log_e \frac{d+\sqrt{d^2-b^2}}{b}}$$

where d is equal to $\frac{y}{2}$ and b is the radius of the plates, viz., 2.5 cm. Thus all the figures in column 23 except the first two may be said to be extrapolated from them. Columns 21 and 22 are similar to columns 18 and 19 respect-

* It is to be noted that in column 5 the fundamental maximum is not exactly at 150.00. For values of y less than 12 cm. this is to be explained by slight inequalities in the lengths of the wires, e. g., BM, HK, etc., leading to the plates together with slight inequalities in the couplings at the four sets of plates. For values of $y=15$ cm. say and above, at which the fundamental tends to split in two the maximum is always at a length slightly greater than 150 as the dotted line of Curve II, Figure 5 shows. In column 19 the value of l_s given for the fundamental is given as if the peak were exactly at 150, i. e., $l=p-150$. If the value of the position of the fundamental given in column 5 were used l_1 would be slightly different from the figure given for it in column 19, being in general slightly less than such figure. This would tend to raise slightly the value of the first figure in column 20 for sets 3 and 6 to 10 inclusive and to lower it in set 5.

ively using the figures of column 23 instead of column 20. In column 21 the first figure in each of the first three sets is arbitrarily taken equal to the first figure in column 18, the first figure in column 22 being necessarily equal to that in column 19. The justification for this may be seen in Figure 7 where columns 18 and 21 are plotted against the frequency number for the first five sets, column 21 being represented by the dotted lines. For $y=8$ the agree-

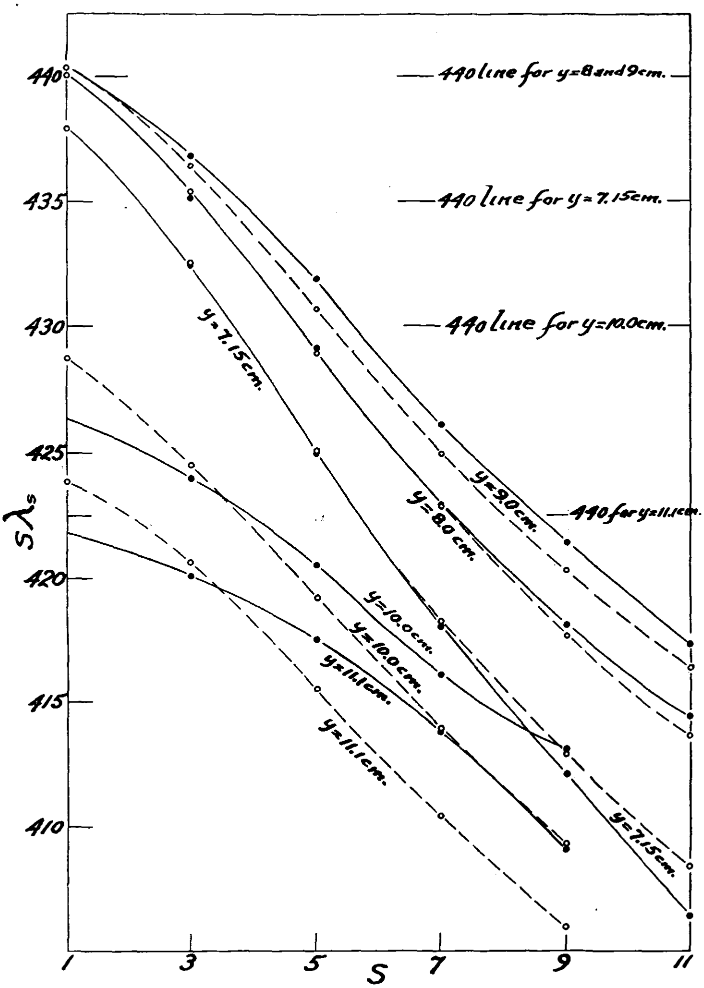

Figure 7.

ment between the two columns is seen to be very good and fairly good for $y=7.15$, the variation being in the opposite direction. For $y=9$ the variation is greater and in the same direction as for $y=8$ cm. For $y=10$ and 11 cm. it is plain that the theoretical and experimental curves can no longer be extrapolated to the same value. Plainly the agreement gets worse as y increases, due to the lack of constancy in column 20 for a given set. In column 24 is given the percent. of error in column 20 as against the value given in column 23.

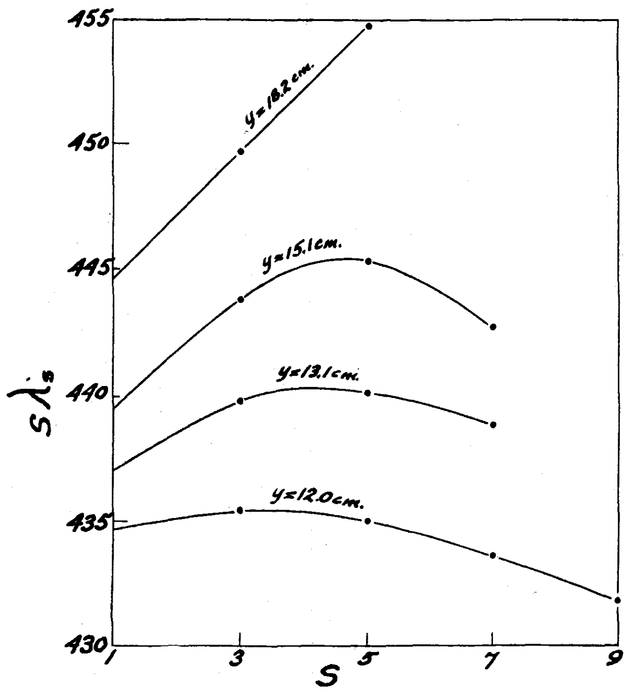

Figure 8.

The lower set of figures in columns 11 to 16 inclusive is obtained by calculating λ_s from the equation given above, using the values for κ_0/κ given in column 23. Since the equation is transcendental the method of approximation has to be used. Multiplying this λ_s (theoretical) by 's' gives column 21. Comparing the two sets of figures in columns 11 to 16 inclusive we get the per cents of error given in column 25.

Remembering that the thing that is experimentally determined is λ_s the percents of error in column 25 are remarkably small. Averaging for all the sets we may say approximately that one per cent. variation in λ_s with its consequent variation in l_s according to the formula

$$l_s = p - \left\{ 150 + (s - a) \frac{\lambda_s}{2} \right\}$$

where s is the frequency number 1, 3, 5, 7, etc., and a the corresponding natural number 1, 2, 3, 4, etc., changes the value of κ_0/κ for values of $s = 1$, 3, 5, 7, 9, 11, respectively 8, 11, 15, 22, 32, 44 per cent. This serves again to emphasize the importance of an accurate measurement of λ_s.

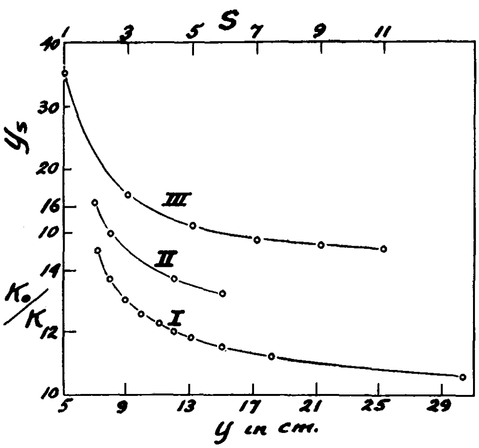

Figure 9.

We have gone into the method of these calculations with some detail because of the dependence placed upon the figures in the theoretical paper by one of us that follows this paper.

Two things stand out from this discussion of Table I. The value of κ_0/κ is very constant for small values of y; and a small error in the wave length determination makes a large error in the constancy of κ_0/κ, the errors being larger proportionately for the smaller wave-lengths. The first point was made by Blake and Sheard and we thus confirm it. For the first three sets of the table the average error in λ_s is only 0.13 per cent.

Blake and Sheard showed that the inconstancy of κ_0/κ for larger values of y was due to the effect of the phase changes introduced by the factor which they called "ϕ (y)" due to the fact that a portion of the wires was at right angles to their main lengths and perhaps to the fact that at points like B and K, Figure 1, there is a change of diameter of the wire where it enters the small rod to which the plate is attached. It is conceivable that the reflection coefficient at such points is not necessarily negligible for the higher harmonics whereas it might be for the lower tones. The greater y is the larger the phase changes for the higher harmonics in comparison with the lower harmonics. These phase changes act in the same direction as the phase change γ due to the end capacities, hence the apparent κ_0/κ would be larger for the higher tones.

If we plot the values of κ_0/κ given in column 23 against y we get Curve I of Figure 9. For the sake of comparison the values Blake and Sheard obtained for plates of the same diameter, but of thickness 4.5 mm. instead of 1 mm. are shown in Curve II. Thus, the extra capacity of the thicker plates for the coupling 11 cm. is about 10 per cent.

SUMMARY.

We have determined experimentally the relation between the tone intensity and the distance between the plates edge-on and have found the optimum distance y_s for a tone of a given frequency number s to fulfill the empirical relation $y_s = Ce^{-\sqrt[4]{s}}$ A theory to account for the tone-intensity as a function of the phase change γ and the wave-length λ is given in the next paper.

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